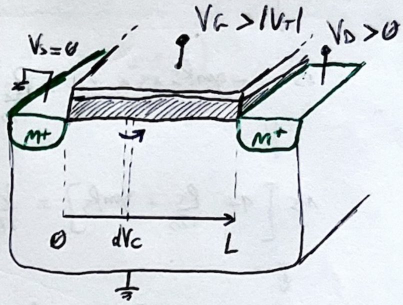
MOSFET FIGURES OF MERIT

Let's consider a MOSFET n-type with source and bulk shorted to ground, while the gate is biased at $V_G > V_T$. The drain is biased with $V_D > 0$. In the channel we have

$$dV_C = I_{DS}\cdot dR = I_{DS}\cdot \frac{dx}{q n \mu_n W \Delta} = I_{DS}\cdot \frac{dx}{Q'_n(x)\,\mu_n W}$$

where $\quad Q'_n(x) = C'_{ox}(V_G - V_C - V_T)$



$$\int_0^{V_{DS}} W \mu_n C'_{ox}(V_G - V_C - V_T)\, dV_C = \int_0^L I_{DS}\, dx$$

$$\downarrow$$

$$I_{DS} = \mu_n C'_{ox}\frac{W}{L}\left[(V_G - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$I_{DS}^{SAT} = \frac{1}{2}\mu_n C'_{ox}\frac{W}{L}(V_{GS} - V_T)^2$$

For V_{DS} values $> V_{DS}^{SAT} = (V_{GS} - V_T)$ the previous relation is no more valid. At steady state the current leaving the source is equal to the one reaching the drain side, so the drift velocity (and the electric field) must increase as we move towards the drain, since $n \downarrow$ and $n \cdot v \simeq$ CONSTANT. In the point called PINCH-OFF we have $Q'_n \simeq 0$ (negligible) and it is expected to move backwards as V_{DS} increases above V_{DS}^{SAT}, while in the area behind the pinch-off point the previous relation is still valid, so

$$I_{DS}^{SAT'} = K'\frac{W}{L'}(V_G - V_T)^2 \qquad \left[K' = \frac{1}{2}\mu_n C'_{ox}\right]$$

where $L' < L$, so $I_{DS}^{SAT'} > I_{DS}^{SAT}$. Assuming that $(L - L')$ remains small, as suggested by the experimental I-V curves, we get that

$$I_{DS} = I_{DS}^{SAT} + \left(\frac{\partial I_{DS}}{\partial V_{DS}}\right)\bigg|_{V_{DS} = V_{DS}^{SAT}}\cdot\left(V_{DS} - V_{DS}^{SAT}\right)$$

where

$$\frac{\partial I_{DS}}{\partial V_{DS}} = \frac{\partial I_{DS}^{SAT}}{\partial L}\,\frac{\partial L}{\partial V_{DS}^{SAT}} = -\frac{I_{DS}^{SAT}}{L}\cdot\frac{\partial L'}{\partial V_{DS}}\bigg|_{V_{DS} = V_{DS}^{SAT}}$$

so

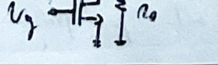
$$I_{DS} = I_{DS}^{SAT}\left[1 + \lambda\left(V_{DS} - V_{DS}^{SAT}\right)\right] \qquad \left(\lambda = -\frac{1}{L}\frac{\partial L'}{\partial V_{DS}}\bigg|_{V_{DS} = V_{DS}^{SAT}}\right)$$

We define $\frac{1}{\lambda} = V_A$ EARLY VOLTAGE.

Does not depend on current!

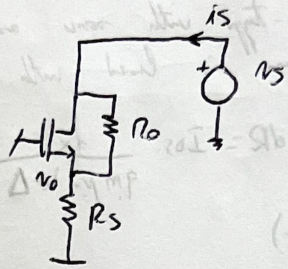
→ GAIN

The maximum gain of a MOSFET is $g_m r_0 = \mu = \frac{2I}{V_{ov}}\cdot\frac{V_A}{I} = \frac{2V_A}{V_{ov}}$



→ RESISTANCES

• R_D



$$v_0 = i_s \cdot R_S$$

$$i_s = -g_m v_0 + \frac{v_s - v_0}{r_0}$$

$$\downarrow$$

$$i_s = -g_m R_S i_s + \frac{v_s}{r_0} - \frac{R_S}{r_0} i_s$$

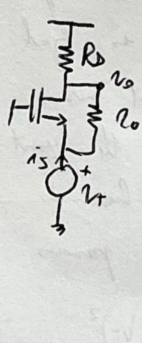
$$\downarrow$$

$$i_s \left[1 + \frac{R_S}{r_0} + g_m R_S \right] = \frac{v_s}{r_0}$$

$$\Downarrow$$

$$\frac{v_s}{i_s} = r_0 + R_S + g_m R_S r_0 = r_0 + R_S \left[1 + g_m r_0 \right]$$

• R_S



$$v_0 = i_s \cdot R_D$$

$$i_s = g_m v_T + \frac{v_T - v_0}{r_0}$$

$$\downarrow$$

$$i_s = -\frac{R_D}{r_0} i_s + \frac{v_T}{r_0} + g_m v_T$$

$$\downarrow$$

$$v_T \left[g_m + \frac{1}{r_0} \right] = i_s \left[1 + \frac{R_D}{r_0} \right]$$

$$\downarrow$$

$$\frac{v_T}{i_s} = \frac{1 + \frac{R_D}{r_0}}{\frac{1}{r_0} + g_m} = \frac{r_0 + R_D}{1 + g_m r_0}$$

→ WEAK INVERSION

In this region electrons are not dominating the electrostatics in the channel, so the bands are almost flat. The concentration of electrons at the source side is computed by using the M-B statistics

$$n(0) = N_S \cdot e^{-\frac{q\phi_B}{kT}}$$

being $\phi_B = V_{BI} - \psi_S$. The current is due to diffusion!

$$J_n = q D_n \frac{dn}{dx} = q D_n \frac{n(0)}{L} = q D_n \frac{n_i^2}{L N_A} e^{\frac{q\psi_S}{kT}}$$

It can be shown that

$$I_{DS} = 4n \left(\frac{1}{2} \mu_n \frac{W}{L} C'_{ox} V_{TH}^2 \right) e^{\frac{q(V_{GS} - V_T)}{nkT}} \qquad \left(n = 1 + \frac{C_{dep}}{C_{ox}} \simeq 1{,}5 \right)$$

so $\frac{\partial I_{DS}}{\partial V_{GS}} = g_m = \frac{I_{DS}}{nV_{TH}}$ and $\mu = g_m r_0 = \frac{V_A}{nV_{TH}}$.

→ MODERATE INVERSION

EKV model

$$IC = \frac{I_D}{4n\left[\frac{1}{2}\mu_n C'_{ox}\frac{W}{L}(V_{TH})^2\right]}$$

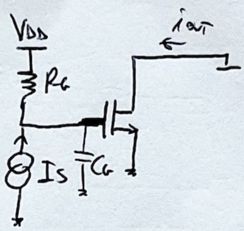
$$g_m = \frac{2}{1+\sqrt{1+4\cdot IC}}\ \frac{I_D}{nV_{TH}}$$

Taxonomy	IC	Bias Range
Weak Inv.	$IC \leq 0,1$	$V_{GS} \leq \lvert V_T\rvert - 0,1V$
Moderate Inv.	$0,1 \leq IC \leq 10$	$\lvert V_T\rvert - 0,1V \leq V_{GS} \leq \lvert V_T\rvert + 0,2V$
Strong Inv.	$IC \geq 10$	$V_{GS} \geq \lvert V_T\rvert + 0,2V$

$$\left(n = 1 + \frac{C_{dep}}{C'_{ox}} = 1,5\right)$$

→ CUT-OFF FREQUENCY

Frequency for a unity current gain



$$\frac{i_{OUT}}{i_S} = g_m R_G$$

$$f_P = \frac{1}{2\pi}\ \frac{1}{R_G \cdot C_G}$$

$$\Downarrow$$

$$f_T = g_m \cancel{R_G} \cdot \frac{1}{2\pi C_G \cdot \cancel{R_G}} = \frac{g_m}{2\pi C_G}$$

Assuming that $C_G = C_{gs} \simeq C'_{ox} \cdot WL$ then

$$f_T \simeq \frac{\cancel{2}\cdot\frac{1}{\cancel{2}}\mu_n \cancel{C'_{ox}}\frac{W}{L}V_{ov}}{2\pi \cancel{C'_{ox}}\,WL} = \frac{1}{2\pi}\ \frac{\mu_n F}{L} = \frac{1}{2\pi}\ \frac{v}{L} = \frac{1}{2\pi}\ \frac{1}{\tau_{DRIFT}}$$

In weak inversion instead, we have that

$$Q' = \frac{q}{2}\cdot n(0)\cdot L \qquad \text{[charge per unit area]}$$

$$J_n = q\,D_n\,\frac{n(0)}{L} \qquad \text{[current density]}$$

$$\tau_{DIFF} = \frac{Q'}{J_n} = \frac{L^2}{2D_n}$$

and $f_T = \dfrac{1}{2\pi\ \tau_{DIFF}}$

$$\tau_{DRIFT} < \tau_{DIFF} \implies \frac{L^2}{\mu_n V_{ov}} < \frac{L^2}{2D_n} \implies V_{ov} > 2\frac{\mu_n}{\mu_n}\frac{kT}{q} = 2V_{TH}$$

Quantitative Description Of Noise

Noise sets constraints on the minimum current value. We can split any signal into

$$y(t) = \underbrace{s(t)}_{\substack{\text{INFORMATION} \\ \Downarrow \\ \text{USEFUL}}} + \underbrace{n(t)}_{\substack{\text{NOISE} \\ \Downarrow \\ \text{UNAVOIDABLE}}} + \underbrace{d(t)}_{\substack{\text{DISTURB} \\ \Downarrow \\ \text{AVOIDABLE}}}$$

We'll consider Gaussian noise (gaussian amplitude distribution), with zero mean value and parameters being time-invariant. We consider σ as a reference value to study noise fluctuations. Moreover, we consider <u>ERGODIC</u> noise, whose time average is indeed equal to the ensemble average.
In general, noise, like signals, can be described as the superposition of orthogonal harmonics with suitable amplitudes. Let's consider a signal made of the superposition of two harmonics, its mean square value is:

$$\langle x(t)^2 \rangle = \langle A^2 \sin^2(\omega_1 t + \varphi) + B^2 \sin^2(\omega_2 t + \varphi) + 2AB \sin(\omega_1 t + \varphi)\sin(\omega_2 t + \varphi) \rangle$$

$$\downarrow$$

$$= \frac{A^2}{2} + \frac{B^2}{2}$$

and it is also equal to the variance, being $\langle x(t) \rangle = 0$. Therefore the variance is equal to the sum of the variances of each harmonic contributing to the noise $x(t)$.

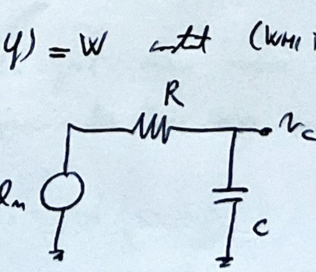
$$\sigma^2 = \sum_i \sigma^2_{f_i} = \int_0^{+\infty} S_n(f)\, df$$

being $S_n(f)$ the <u>POWER SPECTRAL DENSITY</u>.
Let's consider a resistor, whose noise is due to thermal fluctuations of carriers. We may assume that the potential fluctuations have a very broad spectrum, being them made of sequential spikes in the time domain.
It can be concluded that each component in an electronic circuit can be modelled as an equivalent PSD generator (voltage or current), whose effect is transferred to the output and is summed to the other contributions to noise, being them uncorrelated.

→ <u>RESISTORS</u>

As already said, to first order we may assume $S_n(f) = W$ constant (WHITE)
Let's consider this circuit

$$\frac{v_c}{e_n}(s) = \frac{1/sC}{R + \frac{1}{sC}} = \frac{1}{1 + sRC}$$



So

$$\langle v_c^2\rangle = \sigma^2 = \int_0^{+\infty} S_V(f)\,|T(j\omega)|^2\,df = W\cdot\int_0^{+\infty}|T(j\omega)|^2\,df$$

$$= \frac{W}{2\pi\tau}\int_0^{+\infty}\frac{2\pi\tau\,df}{1+(2\pi f\tau)^2} = \frac{W}{2\pi\tau}\left[\arctan(2\pi f\tau)\right]_0^{+\infty} = \frac{W}{2\pi\tau}\cdot\frac{\pi}{2} = \frac{W}{4\tau}$$

we call $\frac{1}{4\tau}$ = ENBW, EQUIVALENT NOISE BAND-WIDTH.

The energy inside the capacitor is

$$\frac{1}{2}C\langle v_c^2\rangle = \frac{kT}{2} \quad \text{[EQUIPARTITION PRINCIPLE]}$$

$$\langle v_c^2\rangle = \frac{kT}{C} = \frac{W}{4RC} \Longrightarrow \boxed{W = 4kTR}$$

→ MOSFETs

- Ohmic region

$$I_{DS} = \frac{1}{2}\mu_n C'_{ox}\left(\frac{W}{L}\right)\left[2(V_{GS}-V_T)V_{DS} - V_{DS}^2\right] \simeq \mu_n C'_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)V_{DS}$$

$$R_{ch} = \frac{1}{G_{ch}} = \frac{1}{\mu_n C'_{ox}\frac{W}{L}V_{ov}} = \frac{1}{g_m}$$ like a resistive channel

$$S_I(f) = \frac{4kT}{R_{ch}} = 4kT g_m$$

- Saturation

$$S_I(f) = 4kT\gamma g_m$$ (γ accounts for the non-uniform channel)

$$\left(\frac{S}{N}\right)^2 = \frac{\frac{\cancel{g_m^2} v_s^2}{2}}{4kT\gamma \cancel{g_m} BW} = \frac{g_m}{8kT\gamma\, BW}\cdot v_s^2$$

⇓ By increasing g_m (current) we increase $\left(\frac{S}{N}\right)^2$, so NOISE sets constraints on current.

INPUT REFERRED NOISE SOURCES

A theorem states that for a TWO-PORT LINEAR NETWORK (characterized by two pairs of input and output terminals) we can substitute the real network with an ideal noiseless one with two equivalent noise sources at the input, whose values are INDEPENDENT OF SOURCE AND LOAD IMPEDANCES.

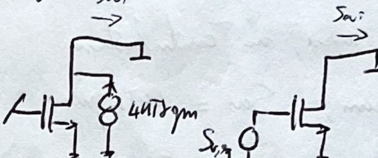
Therefore, to compute $S_{I,eq}$ and $S_{V,eq}$, we may short the output port and take the noise of the short circuit current as output value.

$$\begin{cases} S_{I,eq} \Rightarrow \text{open at the input} \\ S_{V,eq} \Rightarrow \text{short at the input} \end{cases}$$

⟶ MOSFETs

It's a 2-port network, the source being a common terminal for both input and output port.

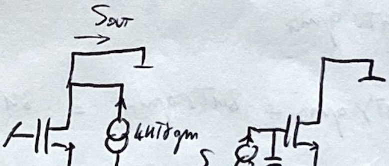
$$\begin{cases} S_{OUT}(f) = 4kT\gamma\, g_m \\ S_{OUT}(f) = S_{V,eq}\, g_m^2 \quad \text{(input shorted)} \end{cases}$$



$$\Downarrow$$

$$S_{V,eq} = \frac{4kT\gamma}{g_m}$$

$$\begin{cases} S_{OUT}(f) = 4kT\gamma\, g_m \\ S_{OUT}(f) = S_{I,eq} \cdot \dfrac{g_m^2}{\omega^2 C_G^2} = S_{I,eq} \cdot \left(\dfrac{\omega_T}{\omega}\right)^2 \end{cases}$$



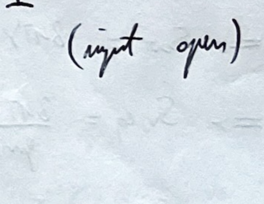
(input open)

$$\Downarrow$$

$$S_{I,eq} = 4kT\gamma\, g_m \left(\frac{\omega}{\omega_T}\right)^2$$

The relative importance of the two generators is set by the source resistance R_S

$$S_{V,IN} = S_{V,eq} + S_{I,eq} \cdot R_S^2 = \frac{4kT\gamma}{g_m}\left[1 + (g_m R_S)^2 \left(\frac{\omega}{\omega_T}\right)^2\right]$$



so for $f \leq \dfrac{f_T}{g_m R_S}$ the current contribution is negligible.

⟶ DIFFERENTIAL STAGE

Strictly speaking, the differential stage is not a two-port linear network, since its output is sensitive to signals on both input terminals. We may write

$$\begin{cases} v_1 = v_{cm} + \dfrac{v_d}{2} \\ v_2 = v_{cm} - \dfrac{v_d}{2} \end{cases} \qquad i_{OUT} = G_{m1} v_1 - G_{m2} v_2 = v_d \left(\frac{G_{m1} + G_{m2}}{2}\right) + v_{cm}(G_{m2} - G_{m1})$$

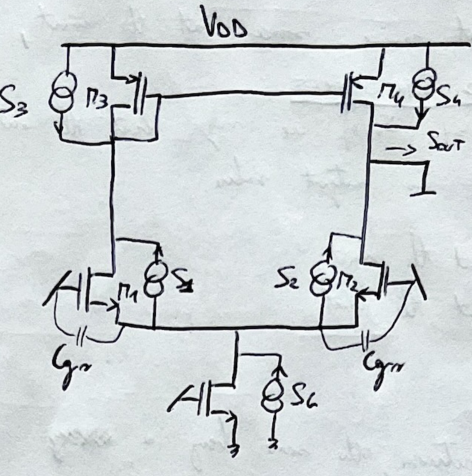
(7)

If we assume $G_{m2} = G_{m1} = G_m$ then $i_{OUT} = G_m v_d$ and the output is only proportional to the input differential voltage, so the OTA becomes a 2-port network.

DEM:

$$S_{OUT} = S_{V,eq} \cdot g_{m1}^2$$

$$S_{OUT} = 4 S_{I,eq} \left(\frac{\omega_T}{\omega}\right)^2$$



i_n flows into the capacitive path at the input, then it causes a differential current in the input pair equal to $\frac{i_n}{sC_{gs}} g_m$, so $i_{OUT} = \frac{2g_m}{sC_{gs}} i_n$

M6: common mode noise, no contribution

M1: splitting theorem, $S_{OUT} = 4kT\gamma g_{m1}$

M2: " " , $S_{OUT} = 4kT\gamma g_{m2}$

M3: $S_{OUT} = 4kT\gamma g_{m3}$

M4: $S_{OUT} = 4kT\gamma g_{m4}$

$$\Rightarrow S_{OUT} = 8kT\gamma g_{mD} + 8kT\gamma g_{mM} = 8kT\gamma g_{mD}\left[1 + \frac{g_{mM}}{g_{mD}}\right] = S_{V,eq}\, g_{mD}^2$$

$$\Rightarrow S_{V,eq} = \frac{8kT\gamma}{g_{mD}}\left[1 + \frac{g_{mM}}{g_{mD}}\right] = \frac{8kT\gamma}{g_{mD}}\left[1 + \frac{V_{OV,D}}{V_{OV,M}}\right]$$

$$\Rightarrow S_{I,eq} = S_{V,eq} \cdot \frac{g_{mD}^2}{4}\left(\frac{\omega}{\omega_T}\right)^2$$

Their relative importance again depends on the source resistance

$$S_{V,IN} = S_{V,eq} + S_{V,eq} \cdot \frac{g_{mD}^2}{4} \cdot R_S^2 \cdot \left(\frac{\omega}{\omega_T}\right)^2$$

so the voltage noise is dominant up to $\omega \sim \frac{2\omega_T}{g_{mD} R_S}$

OISE MODELS

→ THERMAL NOISE (DEM.)

Let's consider two resistors R_0 connected through a lossless coaxial cable, supposed to be isothermal. The cable is sized to match R_0.
The left-hand side resistor causes fluctuations that reach the resistor on the right with no reflections, being them properly matched. The same happens on the opposite direction, and these two flows are equal because of the 2^{nd} principle of thermodynamics. (NO NET TRANSFER OF ENERGY AT TH. EQ.)
Now, let's short the two ends, thus trapping the energy inside. This energy is distributed along the characteristic modes inside the cable, solution of the wave equation (with boundary conditions on the voltage):

$$\lambda_n = \frac{2L}{n} \qquad f_n = \frac{c}{2L} n \qquad f_n \lambda_n = c \qquad \text{[WAVE FUNCTION]}$$

so, in a frequency interval df we have

$$\frac{df}{\frac{c}{2L}} \qquad (\bar{B} \text{ and } \bar{E} \text{ degrees of freedom})$$

modes, each of them having two degrees of freedom, so for the equipartition principle, kT, so

$$dE = \frac{2L}{c} kT \cdot df$$

The two energy flows are equal to

$$E = 2 \cdot P \cdot T = \overbrace{2 \cdot \left(\frac{V_m}{2}\right)^2 \cdot \frac{1}{2R_0}}^{P} \cdot \frac{L}{c} \qquad \left[P \triangleq \text{average power due to one resistor}\right]$$

Considering the harmonics in the df interval, we get

$$dE = S_v \cdot \cancel{df} \, \frac{1}{2R_0} \cdot \frac{\cancel{L}}{\cancel{c}} = \frac{2\cancel{L}}{\cancel{c}} kT \cancel{df} \qquad \underbrace{\left[\frac{V_m^2}{2} = \langle v_m^2 \rangle = S_v \cdot df\right]}_{\text{Harmonic signal}}$$

$$\downarrow$$

$$S_v = 4kT R_0$$

→ JUNCTION SHOT NOISE

The average number of electrons crossing a junction in a pn-junction is represented by the average current, while the actual number per unit time is subjected to fluctuations. It can be compared to an electron being ejected from one plate of a capacitor and reaching the other. This charge causes an opposite charge on the two plates proportional to its distance from it:

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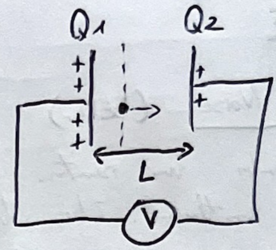
$$\begin{cases} Q_1 = q\,\dfrac{(L-x)}{L} \\ Q_2 = q\,\dfrac{x}{L} \end{cases} \qquad Q_1 + Q_2 = q$$



This induced charge causes a current

$$\frac{dQ_2}{dt} = \left|\frac{dQ_1}{dt}\right| = \frac{q}{L}\cdot v(t) \Rightarrow \text{instantaneous carrier speed}$$

This velocity is

- increasing proportionally to the constant electric field between the plates in the case of the charge between the capacitor's plates;
- constant if we assume the carriers to move in a semiconductor at $v = v_{SAT}$;

in both cases, we get $\int_0^T i(t)\,dt = q$, being it a discrete pulse of a constant duration T.

We can describe the current in a junction as the superposition of many discrete current pulses starting at random time instants, namely

$$q\,h(t) \qquad [h(t) \triangleq \text{normalised shape of the pulse}]$$

We call

$$\lambda = \frac{I}{q} \qquad [\text{average number of carriers crossing the junction per-unit time}]$$

Let's consider a current measured at $t = \bar{t}$, we can say that it is given by the superposition of all the pulses started at $t < \bar{t}$, so

$$i(t) = q\,h(t_1) + q\,h(t_2) + \dots$$

To switch from discrete to continuous we must weight every contribution for the probability for a pulse to occur between t_1 and $t_1 + dt$, so $\lambda\,dt$

$$\langle i(t)\rangle = \int_0^{+\infty} h(t)\,q\lambda\,dt = q\lambda = I \ ✓$$

Similarly we can compute the average square value:

$$i^2(t) = [q\,h(t_1) + q\,h(t_2) + \dots]^2 = \underbrace{q^2h^2(t_1) + q^2h^2(t_2) + \dots}_{\text{SQUARED VALUES}} + \underbrace{q\,h(t_1)\,q\,h(t_2) + \dots}_{\text{CROSS PRODUCTS}}$$

$\downarrow$

$$\langle i^2(t)\rangle = \int_0^{+\infty}\lambda q^2 h^2(t)\,dt + \int_0^{+\infty}\!\!\int_0^{+\infty}(q\,h(x)\,\lambda\,dx)(q\,h(y)\,\lambda\,dy) = q^2\lambda\int_0^{+\infty}h^2(t)\,dt + (q\lambda)^2$$

Knowing that the variance of a casual variable is

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$$

we may write

$$\sigma_I^2 = \langle i^2 \rangle - \langle i \rangle^2 = \int_0^{+\infty} h^2(t)\,dt\,(q^2)\lambda + (q\lambda)^2 - (q\lambda)^2 = q^2\lambda \int_0^{+\infty} h^2(t)\,dt$$

$$\downarrow$$

$$= qI \int_0^{+\infty} h^2(t)\,dt$$

Now, using the Parseval theorem, we can write

$$\int_0^{+\infty} |h(t)|^2\,dt = \int_{-\infty}^{+\infty} |H(f)|^2\,df = 2\int_0^{+\infty} |H(f)|^2\,df$$

$h(t)$ real $\Leftrightarrow$ $|H(f)|$ even function

So, in the end, we get

$$\sigma_I^2 = 2qI \int_0^{+\infty} |H(f)|^2\,df = \int_0^{+\infty} S_I(f)\,df = 2q^2\lambda \int_0^{+\infty} |H(f)|^2\,df$$

and from the equation we derive

$$S_I(f) = 2qI\,|H(f)|^2$$

Finally, since our pulses have a duration T of the order of ps, then the amplitude of their Fourier transform will be $\simeq 100\,\text{GHz}$, so to first order we can consider $|H(f)|^2 \simeq 1$ in the BW of interest. ✓

Accounting for both physical mechanism involved in diode's current (DRIFT + DIFFUSION) we get that

$$S_I = 2q\,(I + 2I_0) \qquad \begin{cases} I_{DIFF} = I + I_0 \\ I_{DRIFT} = -I_0 \end{cases}$$

So, we get

REVERSE BIAS $\Rightarrow$ $S_I = 2qI_0$

FORWARD BIAS $\Rightarrow$ $S_I \simeq 2qI$

ZERO BIAS $\Rightarrow$ $S_I = 4qI_0 = 4qkT\dfrac{I_0}{kT} = 4kT\,g_{m,0}$

Finally, we can say that also the current in weak-inversion MOSFETS is affected by shot noise, being it composed by the superposition of current pulses generated by carriers surfing above the barrier.

$$S_I = 2qI_D = 2q\,I_D\cdot\frac{nV_{TH}}{nV_{TH}} = 2qn\frac{kT}{q}\cdot g_m = 4kT\frac{n}{2}g_m = 4kT\gamma\, g_m$$

where $\boxed{\gamma = \frac{n}{2}}$

⟶ <u>RTN</u>

Let's consider a resistor made of n-Si. The average current flowing through it is

$$I = \frac{V}{R} = V\,\frac{W\cdot\Delta}{L}\,q\mu_n n = q\mu_n\cdot\frac{N}{L^2}V \qquad (I\propto N)$$

being N the total number of free carriers in a volume $\mathcal{V} = W\cdot\Delta\cdot L$

If a carrier is trapped or emitted by a trap, this causes a reduction or an increase of the total current. Each event causes a variation of N equal to ΔN, and so

$$\frac{\Delta I}{I} = \frac{\Delta N}{N} \Rightarrow \Delta I = \frac{I}{N} \qquad (\text{for } \Delta N = 1)$$

Each trapping or emission event causes current variations that happen to recover the steady state value exponentially, since the time needed to recover it is a statistical variable. So we can write each pulse like:

$$i(t) = \Delta I\cdot e^{-\frac{t}{\tau}} = (\Delta I\cdot\tau)\cdot\frac{1}{\tau}e^{-\frac{t}{\tau}} = Q\cdot h(t)$$

At steady state λ (# of emission events per unit time) is equal to # of capture events per unit time. We expect

$$\lambda \propto N_T \quad (\text{number of traps in the volume})$$

$$\downarrow$$

$$\lambda = \beta\frac{N_T}{\tau}$$

As already shown for the pn-junction, a noise characterized by the superposition of well-defined current pulses of area Q can be written as

$$S_I(f) = 2\lambda Q^2|H(f)|^2 = 2\lambda Q^2\frac{1}{(1+\omega^2\tau^2)}$$

$$\downarrow$$

$$= 2\beta\frac{N_T}{\tau}\Delta I^2\cdot\tau^2\cdot\frac{1}{1+\omega^2\tau^2} = 2\beta N_T\Delta I^2\frac{\tau}{1+\omega^2\tau^2}$$

$$\downarrow$$

$$= 2\beta N_T\left(\frac{I}{N}\right)^2\frac{\tau}{1+\omega^2\tau^2}$$

This contribution is summed to the one due to emission events, being them statistically independent from the capture events.

so we get

$$S_I(f) = 4\beta N_T \left(\frac{I}{N}\right)^2 \underbrace{\frac{\tau}{(1+w^2\tau^2)}}_{\text{Lorentzian shape}}$$

where

$$\underbrace{\beta = \frac{\tau_E\tau_C}{(\tau_E+\tau_C)^2}} \qquad \tau = \frac{\tau_E\tau_C}{\tau_E+\tau_C}$$

asymmetry coefficient, maximum for $\tau_E = \tau_C$ and equal to $\frac{1}{4}$ given by states at the Fermi level

$$S_I(f) = N_T \cdot \left(\frac{I}{N}\right)^2 \frac{\tau}{1+w^2\tau^2}$$

⟶ 1/f NOISE

Defects with different time constants may exist, so let's introduce the function $g(\tau)$ which is a distribution of defects over the τ axis, so that

$$dN_T(\tau) = N_T\, g(\tau)d\tau \qquad (g(\tau) \text{ is normalized})$$

therefore we can write

$$S_I(f) = N_T \cdot \left(\frac{I}{N}\right)^2 \int_{\tau_{MIN}}^{\tau_{MAX}} \frac{g(\tau)\,\tau}{1+w^2\tau^2}\, d\tau$$

$\begin{cases} \tau_{MIN} = \text{shortest } \tau \text{ that can be measured, set by the BW of the apparatus} \\ \tau_{MAX} = \text{set by the experiment duration} \end{cases}$

McWhorter pointed out that in a MOSFET free carriers can be captured by defects at the interface and inside it, due to tunneling. Defects inside the oxyde have a

$$\tau = \tau_0 e^{\gamma x} \qquad \begin{cases} \gamma \propto \text{height of the barrier} \\ \tau_0 = \tau \text{ for defects at the interface} \end{cases}$$

Let's consider defects to be distributed uniformly all over the oxyde thickness

$$n_T' = \frac{N_T}{t_{ox}} \quad [\text{density per unit length}]$$

so we can write

$$n_T'\,dx = N_T \frac{dx}{t_{ox}} = N_T\, g(\tau)\, d\tau$$

and

$$d\tau = \gamma\tau_0 e^{\gamma x} dx = \gamma\tau\, dx \implies dx = \frac{d\tau}{\gamma\tau}$$

so

$$g(\tau) = \frac{1}{\gamma\tau t_{ox}} \qquad (\text{we've linked } g(\tau) \text{ to other parameters we know})$$

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$$\Rightarrow S_I(f) = \frac{N_T}{\gamma t_{ox}} \left(\frac{I}{N}\right)^2 \int_{\tau_{MIN}}^{\tau_{MAX}} \frac{d\tau}{1+w^2\tau^2} = \frac{N_T}{\gamma t_{ox}} \frac{1}{w} \left(\frac{I}{N}\right)^2 \left[\arctan(w\tau)\right]_{\tau_{MIN}}^{\tau_{MAX}}$$

$$= \frac{N_T}{\gamma t_{ox}} \frac{1}{w} \left(\frac{I}{N}\right)^2 \left[\arctan(w\tau_{MAX}) - \arctan(w\tau_{MIN})\right]$$

Considering $w\tau_{MAX} >> 1$ and $w\tau_{MIN} << 1$ we get

$$S_I(f) = \frac{N_T}{\gamma t_{ox} w} \left(\frac{I}{N}\right)^2 \frac{\pi}{2} = \frac{N_T}{4\gamma t_{ox}} \left(\frac{I}{N}\right)^2 \cdot \frac{1}{f}$$

$\longrightarrow$ TSIVIDIS FORMULA

$N = C'_{ox} (WL)(V_G - V_T) \cdot \frac{1}{q}$ carriers in the channel

$N_T = n_T \cdot WL\, t_{ox}$ charges in the oxyde

$$S_I(f) = \frac{n_T \cdot WL \cdot t_{ox}}{\gamma t_{ox} \cdot 4} q^2 \frac{\left[\frac{1}{2}\mu_n C'_{ox}\left(\frac{W}{L}\right)(V_G - V_T)^2\right]^2}{\left[C'_{ox}(WL)(V_G - V_T)\right]^2} \cdot \frac{1}{f}$$

$$= \frac{n_T \cdot \cancel{W\cdot L}\, q^2}{4 \cdot \gamma (W\cdot L)^{\cancel{2}}} \cdot \frac{1}{f} \cdot \frac{1}{4}\mu_n^2 \cdot \left(\frac{W}{L}\right)^2 \cdot (V_G - V_T)^2 =$$

$$= \frac{n_T q^2}{8\gamma L^2} \cdot \frac{\mu_n}{C'_{ox}} \cdot \frac{I}{f} = \frac{K_I^{1/f}}{L^2} \frac{I}{f}$$

$$S_V(f) = g_m^{-2} S_I(f) = \frac{n_T \mu_n q^2}{8\gamma C_{ox} L^2} \cdot \frac{\cancel{I}}{f} \cdot \frac{V_{ov}^2}{4 \cdot I^{\cancel{2}}} = \frac{1}{f} \frac{n_T}{16\gamma C'_{ox}} \cdot \cancel{\frac{1}{2}\mu_n C'_{ox}\left(\frac{W}{L}\right)} V_{ov}^2 \cdot \frac{1}{\cancel{I}} \cdot \frac{1}{C'_{ox}WL}$$

$$= \frac{1}{f} \frac{n_T q^2}{16\gamma C'_{ox}} \cdot \frac{1}{C'_{ox}WL} = \frac{K_V^{1/f}}{C'_{ox}WL} \cdot \frac{1}{f} \checkmark$$

Prototypical Differential Stage

Differential amplifiers $\Rightarrow$ amplify the potential difference, regardless of the average potential

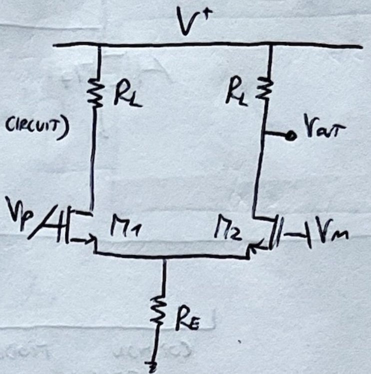
$CMRR \triangleq \left|\frac{G_d}{G_{cm}}\right|$

1 Resistances

$$G_d = \frac{g_m R_L}{2} = \frac{I R_L}{V_{ov}}$$

$$G_{cm} = \frac{-v_{cm}}{2R_E + \frac{1}{g_m}} \cdot R_L \simeq -v_{cm}\frac{R_L}{2R_E} \quad \text{(HALF CIRCUIT)}$$

$$CMRR = \frac{g_m R_L}{2} \cdot \frac{2R_E}{R_L} = g_m R_E$$



(SINGLE ENDED OUTPUT)

- $G_{d,MAX}$ limited by $I \cdot R_L = V_{L\_MAX}$ (M_1 & M_2 may become ohmic)
- $CMRR|_{MAX}$ limited by the maximum current I, which is limited by the maximum V_{LOAD}.

2 Current Generators

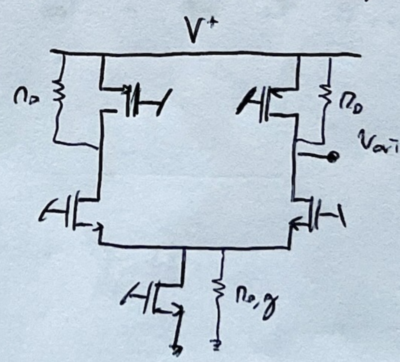
We may think to improve R_{out} without having to deal with high voltage drops by replacing R_L with current generators.
Similarly, we should also increase R_E by substituting it with a current generator.

$$G_{cm} = \frac{r_0}{2r_{0,g}}$$

$$G_d = \frac{g_m r_0}{2}$$

$$CMRR = \frac{g_m r_0}{2} \cdot \frac{2r_{0,g}}{r_0} = g_m r_{0,g}$$

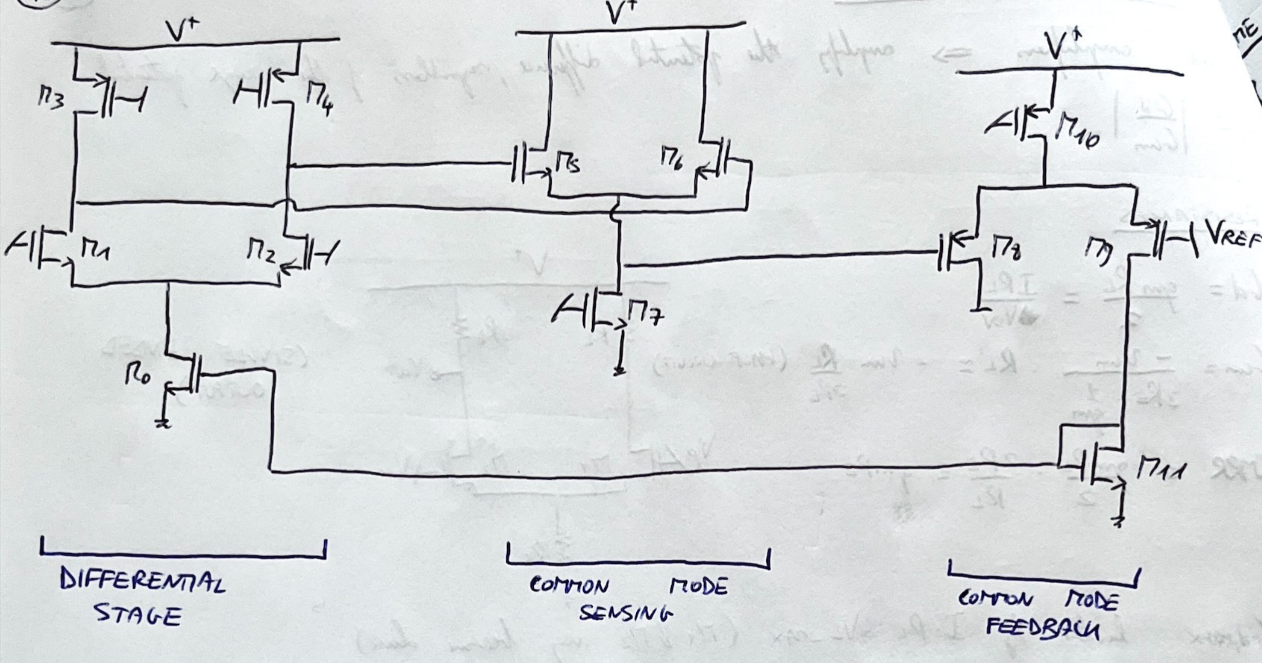
Maybe improve $r_{0,g}$ using CASCODES!



3 Common Mode Feedback

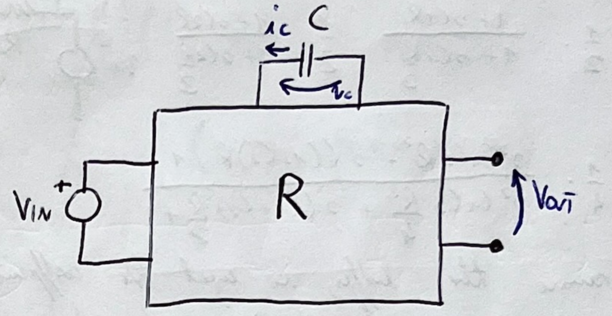
The previous circuit indeed needs to have the pMOS transistors perfectly matching the nMOS current, otherwise the intermediate drain nodes would act in order to push the upper or lower pair into ohmic region, until the currents match. Ohmic transistors though have lower r_0, so lower G_d $\Rightarrow$ BAD!

We need feedback:


V+
M3
M4
M1
M2
M0
V+
M5
M6
M7
V+
M10
M8
M9
VREF
M11
DIFFERENTIAL STAGE
COMMON MODE SENSING
COMMON MODE FEEDBACK

TIME CONSTANTS METHOD

Let's consider a linear time-invariant network with a resistive core and one capacitor:



$$\begin{cases} \text{INPUT VARIABLES: } i_C \;\&\; v_{IN} \\ \text{OUTPUT VARIABLES: } V_{OUT} \;\&\; V_C \end{cases}$$

$\Downarrow$

$$\begin{cases} v_{OUT} = A_0 v_{IN} + R_m i_C \\ v_C = B_0 v_{IN} + R_1 \cdot i_C \end{cases}$$

$(A_0, B_0, R_m, R_1) \in \mathbb{R}$ being the core resistive.

Let's consider that $v_C = -\dfrac{i_C}{sC}$

$\Downarrow$

$$\begin{cases} v_{OUT} = A_0 v_{IN} - sCR_m v_C \\ v_C = B_0 v_{IN} - sCR_1 v_C \end{cases} \implies v_C = \frac{B_0 v_{IN}}{1+sCR_1} \;;\; v_{OUT} = A_0 v_{IN}\left[1 - sC\,\frac{R_m B_0/A_0}{1+sCR_1}\right]$$

$$v_{OUT} = A_0 v_{IN}\,\frac{1+sC(R_1 - R_m B_0/A_0)}{1+sCR_1}$$

So we have a pole at $s = -\dfrac{1}{CR_1}$ and a zero at $s = -\dfrac{1}{C(R_1 - R_m B_0/A_0)}$

For a zero to exist it must be

$$A_0 v_{IN} + R_m i_C = 0$$

$\downarrow$

$$i_C = -\frac{A_0}{R_m} v_{IN} \quad \text{or} \quad v_{IN} = -\frac{R_m}{A_0} i_C$$

So

$$v_C = -\frac{B_0}{A_0} R_m i_C + R_1 i_C$$

$\downarrow$

$$\frac{v_C}{i_C} = \left(R_1 - \frac{R_m B_0}{A_0}\right) = R_{01} \qquad \text{[RESISTANCE SEEN ACROSS C when } V_{OUT}=0\text{]}$$

$$\implies \frac{v_{OUT}}{v_{IN}} = A_0\,\frac{1+sCR_{01}}{1+sCR_1}$$

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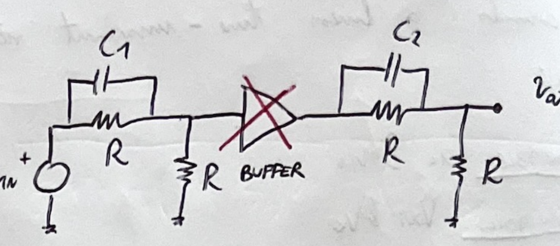
Let's consider now two RC networks decoupled by a buffer, the overall T.F. will be the cascade of the two:



$$T(s) = \frac{1}{2} \cdot \frac{1+sC_1R}{1+sC_1\frac{R}{2}} \cdot \frac{1}{2} \cdot \frac{1+sC_2R}{1+sC_2\frac{R}{2}}$$

$$\downarrow$$

$$= \frac{1}{4} \cdot \frac{s^2C_1C_2R^2 + s\left((C_1+C_2)R\right) + 1}{s^2C_1C_2\frac{R^2}{4} + s\left((C_1+C_2)\frac{R}{2}\right) + 1}$$

If we remove the buffer, we expect the coefficients to change, but just for the resistive part. Moreover, also the DC gain changes!

[$A_0 \triangleq$ GAIN WITH ALL THE CAPACITORS OPEN]

$$T(s) = A_0 \frac{s^2C_1C_2\alpha_{12} + s(C_1\alpha_1 + C_2\alpha_2) + 1}{s^2C_1C_2\beta_{12} + s(C_1\beta_1 + C_2\beta_2) + 1}$$

① $C_2 \to \emptyset$

$$T(s) = A_0 \frac{sC_1\alpha_1 + 1}{sC_1\beta_1 + 1}$$

and we know that $\alpha_1 = R_{1,0}^{(0)}$ $\quad \beta_1 = R_1^{(0)}$

② $C_1 \to \emptyset$

$$T(s) = A_0 \frac{sC_2\alpha_2 + 1}{sC_2\beta_2 + 1}$$

and we know that $\alpha_2 = R_{2,0}^{(0)}$ $\quad \beta_2 = R_2^{(0)}$

③ $C_1 \to \infty$ (a short)

$$T(s) = A_0 \frac{s^2C_1C_2\alpha_{12} + sC_1R_{01}^{(0)}}{s^2C_1C_2\beta_{12} + sC_1R_1^{(0)}} = A_0 \frac{sC_1C_2\alpha_{12} + C_1R_{01}^{(0)}}{sC_1C_2\beta_{12} + C_1R_1^{(0)}} =$$

$$= A_0 \frac{R_{01}^{(0)}}{R_1^{(0)}} \frac{sC_2\alpha_{12}/R_{01}^{(0)} + 1}{sC_2\beta_{12}/R_1^{(0)} + 1}$$

and since the network can be seen as the one of a single capacitor C_2 and C_1 as a short, we may write

$$\begin{cases} \dfrac{\alpha_{12}}{R_{01}^{(0)}} = R_{02}^{(1)} \\ \dfrac{\beta_{12}}{R_1^{(0)}} = R_2^{(1)} \end{cases} \Rightarrow \begin{cases} \alpha_{12} = R_{01}^{(0)}R_{02}^{(1)} \\ \beta_{12} = R_1^{(0)}R_2^{(1)} \end{cases}$$

If we chose $C_2 \to \infty$ instead of C_1, we'd get

$$\begin{cases} \alpha_{12} = R_{02}^{(0)}R_{01}^{(1)} \\ \beta_{12} = R_2^{(0)}R_1^{(2)} \end{cases}$$

which must be equal to the one obtained for $C_1 \to \infty$, since the T.F. is unique!

Popular Results

Starting from a 3^{RD} order T.F., we may write

$$D(s) = b_3 s^3 + b_2 s^2 + b_1 s + 1 = \left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right)\left(1 - \frac{s}{p_3}\right)$$

$$b_1 = -\left(\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3}\right) \simeq -\frac{1}{p_1} \qquad (\text{assuming } p_1 << p_2, p_3)$$

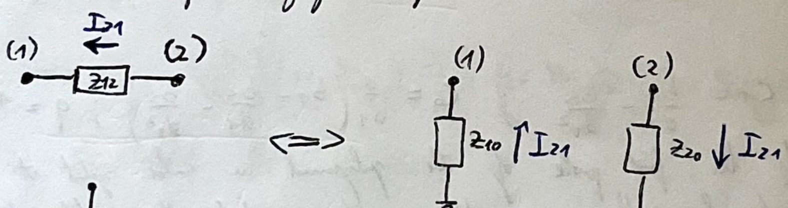
@ HF

$$D(s) \simeq b_3 s^3 + b_2 s^2 = s^2 (b_3 s + b_2) = 0$$

$$p_3 \simeq -\frac{b_2}{b_3} = \text{reshape terms} \dots$$

Miller Theorem

We want to replace Z_{12} with two impedances Z_{10} and Z_{20}, we must set that the current entering or going out of the nodes remains unchanged



$$I_{21} = \frac{V_2 - V_1}{Z_{12}} = -\frac{V_1}{Z_{10}} \qquad \left[\frac{V_2}{V_1} = H(s)\right] \qquad I_{21} = \frac{V_2 - V_1}{Z_{12}} = \frac{V_2}{Z_{20}}$$

$$\downarrow$$

$$V_2 - V_1 = -\frac{Z_{12}}{Z_{10}} V_1 \qquad Z_{20} = \frac{V_2}{V_2 - V_1} Z_{12}$$

$$\downarrow$$

$$Z_{10} = -\frac{V_1}{V_2 - V_1} \cdot Z_{12} = \frac{V_1}{V_1 - V_2} Z_{12} \qquad Y_{20} = \frac{V_2 - V_1}{V_2} Y_{12}$$

$$\downarrow$$

$$\Rightarrow Y_{10} = \frac{V_1 - V_2}{V_1} Y_{12} = [1 - H(s)] Y_{12} \qquad \Rightarrow Y_{20} = \frac{H(s) - 1}{H(s)} Y_{12}$$

Approximations and Circuit Insights

It's important to derive some estimates of the poles and zeros by using approximations in order to understand which parameters are limiting the stability. Let's consider a third order T.F. for example:

$$D(s) = b_3 s^3 + b_2 s^2 + b_1 s + 1$$

We may divide it for a first order term

$$(b_3 s^3 + b_2 s^2 + b_1 s + 1) \div (d_1 s + 1)$$

$$
\begin{array}{l|l}
b_3 s^3 + b_2 s^2 + b_1 s + 1 & d_1 s + 1 \\
\hline
b_3 s^3 + \frac{b_3}{d_1} s^2 & \frac{b_3}{d_1} s^2 + \frac{1}{d_1}\left(b_2 - \frac{b_3}{d_1}\right) s + \frac{1}{d_1}\left(b_1 - \frac{b_2}{d_1} - \frac{b_3}{d_1^2}\right) \\
\hline
// \left(b_2 - \frac{b_3}{d_1}\right) s^2 + b_1 s + 1 & \\
\left(b_2 - \frac{b_3}{d_1}\right) s^2 + \frac{1}{d_1}\left(b_2 - \frac{b_3}{d_1}\right) s & \\
\hline
// \left(b_1 - \frac{b_2}{d_1} - \frac{b_3}{d_1^2}\right) s + 1 & \\
\left(b_1 - \frac{b_2}{d_1} - \frac{b_3}{d_1^2}\right) s + \frac{1}{d_1}\left(b_1 - \frac{b_2}{d_1} - \frac{b_3}{d_1^2}\right) &
\end{array}
$$

So we get

$$c_2 s^2 + c_1 s + c_0 + \frac{q}{d_1 s + 1}$$

where

$$c_2 = \frac{b_3}{d_1} \quad ; \quad c_1 = \frac{b_2}{d_1} - \frac{b_3}{d_1^2} \quad ; \quad c_0 = \frac{1}{d_1}\left(b_1 - \frac{b_2}{d_1} - \frac{b_3}{d_1^2}\right) \quad ; \quad q = 1 - \frac{1}{d_1}\left(b_1 - \frac{b_2}{d_1} - \frac{b_3}{d_1^2}\right)$$

If $-\frac{1}{d_1}$ was exactly the first pole of the polynomial we could split it into a 2nd and a 1st order polynomial with $q = 0$, thus simplifying a lot the problem. However, even if we don't know exactly p_1, we know that it is $\simeq$ set by the Miller capacitance, so we may say

$$p_1 \simeq -\frac{1}{b_1}$$

so we get

$$c_2 = \frac{b_3}{b_1} \quad ; \quad c_1 = \frac{b_2}{b_1} - \overbrace{\frac{b_3}{b_1^2}}^{\text{Usually negligible}} \quad ; \quad c_0 = \frac{1}{b_1}\left[b_1 - \overbrace{\frac{b_2}{b_1} - \frac{b_3}{b_1^2}}^{\text{Usually negligible}}\right] \quad ; \quad q = \overbrace{\frac{1}{b_1}\left(\frac{b_2}{b_1} - \frac{b_3}{b_1^2}\right)}^{\text{Usually negligible}}$$

and split the denominator like this

$$(c_2 s^2 + c_1 s + 1)(1 + s b_1)$$

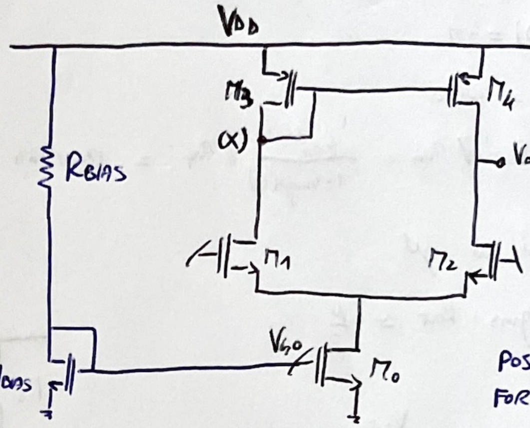
assuming $q \simeq 0$. It can be shown that these results are really close to the ones obtained by shorting C_c and computing the remaining poles using the extended time constant method.

SINGLE - ENDED OTA

The single - ended configuration is another option to guarantee consistent bias of the stage. The adoption of the current mirror binds the current from M_4 to the current delivered by the tail, hence implementing the control loop directly in the differential stage.

In order for the bias to be consistent, we need

$$\begin{cases} I_1 = I_3 \\ I_2 = I_4 \end{cases}$$



POSSIBLE CIRCUIT FOR $V_{G,0}$

(neglecting V_{ov} in $(V_{DS}-V_{ov})/V_A$)

$$\begin{cases} \frac{1}{2}\mu_n C'_{ox}\left(\frac{W}{L}\right)_1 (V_{ov,D})^2 \left[1+\frac{V_X-V_S}{V_A}\right] = I_1 \\ \frac{1}{2}\mu_p C'_{ox}\left(\frac{W}{L}\right)_3 (V_{ov,M})^2 \left[1+\frac{V_{DD}-V_X}{V_A}\right] = I_3 \\ \frac{1}{2}\mu_n C'_{ox}\left(\frac{W}{L}\right)_2 (V_{ov,D})^2 \left[1+\frac{V_{OUT}-V_S}{V_A}\right] = I_2 \\ \frac{1}{2}\mu_p C'_{ox}\left(\frac{W}{L}\right)_4 (V_{ov,M})^2 \left[1+\frac{V_{DD}-V_{OUT}}{V_A}\right] = I_4 \end{cases}$$

$$\frac{I_1}{I_2} = \frac{I_3}{I_4} \implies \left(1+\frac{V_X-V_S}{V_A}\right)\left(1+\frac{V_{DD}-V_{OUT}}{V_A}\right) = \left(1+\frac{V_{DD}-V_X}{V_A}\right)\left(1+\frac{V_{OUT}-V_S}{V_A}\right)$$

by inspection it turns out that $V_X = V_{OUT}$

→ **VOLTAGE SWINGS**

- $V_{CM}|_{MAX} = V_{DD} - V_{SG,3} + |V_T| = V_{DD} - V_{ov,3} - |V_T| + |V_T| = V_{DD} - V_{ov,3}$ (Saturation of M_1)
- $V_{CM}|_{MIN} = V_{ov,0} + V_{GS,1} = V_{ov,0} + V_{ov,1} + |V_T|$ (Saturation of M_0)

Not symmetric! Towards ground it has to accommodate for a V_{GS} in addition to an overdrive voltage.

- $V_{OUT}|_{MIN} = V_{CM\_MIN} - |V_T| = V_{ov,0} + V_{ov,1}$ (Saturation of M_2)
- $V_{OUT}|_{MAX} = V_{DD} - V_{ov,4}$ (Saturation of M_4)

→ **DIFFERENTIAL GAIN**

Due to the introduction of the mirror, the circuit is not symmetric anymore, since the resistances on the drain of M_1 and M_2 are different! In order to recover symmetry, we use the Norton theorem:

- in order to compute i_{cc} we need to short the drain of M_4. In this way both the drains of M_1 and M_2 are connected to low-Z nodes, we can assume them to be 0 with an error $\sim \frac{1}{g_m R_0}$.

(21)

Now we've restored symmetry and we may say that vdiff splits evenly on the two transistors, thus having the same status as in small signal operation. Neglecting also the mirroring errors, we get

$$i_{cc} = g_m v_{diff}$$

- R_{out} can be computed by noticing that we have a loop acting to fix v_{out} to V.G., so we may cut the loop, reconstruct the impedance ($2r_{o2}$) and evaluate

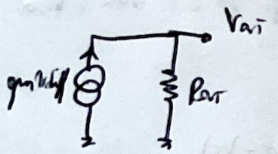
$$G_{loop}(0) = -1$$

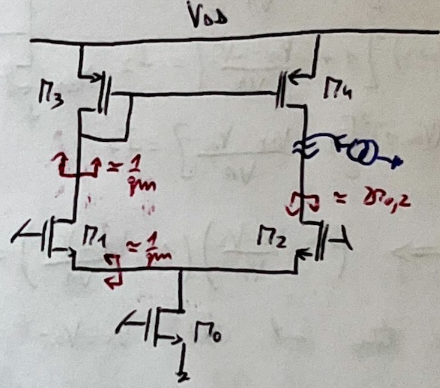
$$R_{out}'^{(0)} = 2r_{o2}$$

$$R_{out} = R_{out}' \,//\, r_{o4} = \frac{R_{out}'^{(0)}}{1-G_{loop}(0)} \,//\, r_{o4} = r_{o2}//r_{o4}$$

In the end we get

$$G_0 = g_{mD} \cdot R_{out} \simeq \frac{\mu}{2}$$



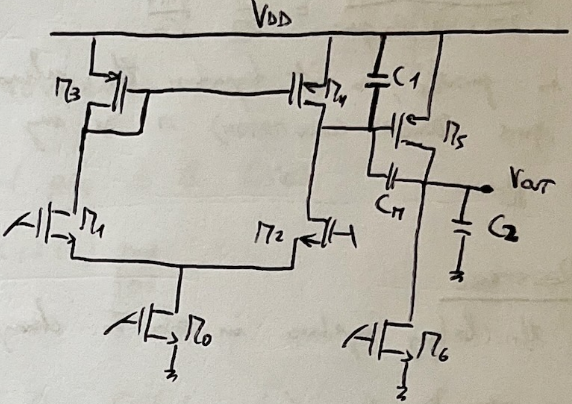


COMPENSATION (TWO-STAGE)

- **MILLER**

Split the two poles due to the two high impedance nodes by placing a C_M bridging the gate-drain nodes of a transistor.

The three capacitors are dependent (they form a loop) so we expect only two poles in the T.F.:



$$b_1 = C_1 \cdot R_1^0 + C_2 R_2^0 + C_M R_M^0$$

$$\begin{cases} R_1^0 = R_{OUT,1} = R_1 \\ R_2^0 = R_{OUT,2} = R_2 \\ R_M^0 = R_1 + g_{m5} R_2 R_1 + R_2 \quad \text{(Miller effect)} \end{cases}$$

$$\Rightarrow \quad b_1 = C_1 R_{OUT,1} + R_2 (C_2 + C_M) + (1 + g_{m5} R_2) C_M R_1 = \tau_1 + \tau_2 + \tau_3$$

$$b_2 = C_1 C_2 R_1^0 R_2^1 + C_1 C_M R_1^0 R_M^1 + C_2 C_M R_2^0 R_M^2$$

$$\begin{cases} R_2^1 = R_2 \\ R_M^1 = R_2 \\ R_M^2 = R_1 \end{cases}$$

$$\Rightarrow \quad b_2 = C_1 R_1 R_2 (C_2 + C_M) + R_1 R_2 C_2 C_M = \tau_1 \tau_2 + \tau_4$$

$$b_3 = C_1 C_2 C_M R_1^0 R_2^1 R_M^{1,2} = 0 \quad \text{(as expected)}$$

We expect also a zero when

$$s C_M v_s = g_{m5} v_s$$
$$\downarrow$$
$$s = \frac{g_{m5}}{C_M} \quad \text{(POSITIVE)}$$

If the roots of the denominator are split of more than a decade, we get

$$p_L \simeq -\frac{1}{b_1} \simeq -\frac{1}{R_1 g_{m5} R_2 C_M}$$

$$p_H \simeq -\frac{b_1}{b_2} \simeq -\frac{R_1 g_{m5} R_2 C_M}{R_1 R_2 C_1 (C_2 + C_M) + R_1 R_2 C_2 C_M} = -\frac{g_{m5} C_M}{C_1 C_2 + C_1 C_M + C_2 C_M}$$

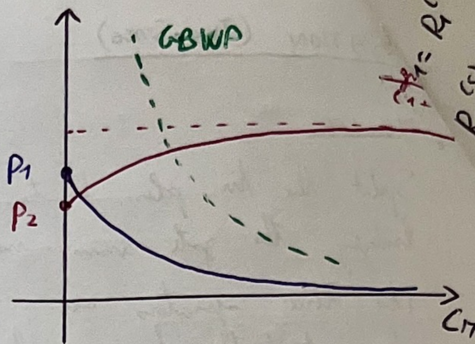
(23)

For large C_M values we get

$$p_H \simeq -\frac{g_{m5}}{C_1 + C_2}$$

and p_L moves to lower frequencies.

Finally, we have



$$GBWP = \frac{g_{m1} R_1 \, g_{m5} R_2}{2\pi \, C_M R_1 R_2 \, g_{m5}} = \frac{1}{2\pi}\frac{g_{m1}}{C_M}$$

The zero is positive, so it degrades the phase margin! We must shift it to HF by increasing g_{m5} (POWER CONSUMPTION) or we may increase C_M in order to move p_L away from p_H.

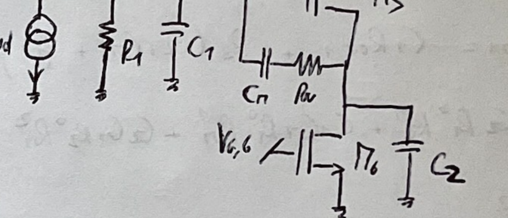
- NULLING RESISTANCE

We modify the bridging impedance in order to change the zero:

$$\frac{v_5}{R_N + \frac{1}{sC_M}} = g_{m5} v_5$$

$$\downarrow$$

$$z = -\frac{1}{C_M\left[R_N - \frac{1}{g_{m5}}\right]}$$



We may size R_N in order to have a negative zero cancelling out with the second pole.

Due to R_N we have three poles

in series to the resistance across gate and drain of M5

$$b_1 = C_1R_1 + \cancel{}\, C_2R_2 + C_C\left(R_1 + R_2 + g_{m5}R_1R_2 + R_N\right)$$

being $R_N \simeq \frac{1}{g_{m5}}$ we expect

$$b_1 \simeq C_M \, g_{m5} R_2 R_1 \qquad \text{(UNCHANGED)}$$

$$\Rightarrow \tau_L \simeq b_1 \;;\; p_L = -\frac{1}{b_1}$$

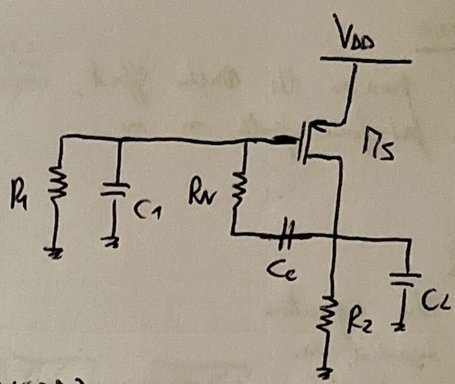
We can compute τ_H using the time constant method:

$$\begin{cases} \tau_1^{\infty} = (R_1 // R_N)\, C_1 \simeq R_N C_1 \\ \tau_2^{\infty} = (R_2 // R_N)\, C_2 \simeq R_N C_2 \\ \tau_C^{\infty} = R_N \end{cases}$$

$$p_H = -\left(\frac{1}{\tau_1^{\infty}} + \frac{1}{\tau_2^{\infty}} + \frac{1}{\tau_3^{\infty}}\right) \simeq -\frac{1}{R_N\,(C_1 // C_2 // C_C)}$$

We can compute the second pole assuming that C_C is a short (this approximation is clearly justified in "APPROXIMATIONS AND CIRCUIT INSIGHTS")

$$b_1 = R_1^{(c)} C_1 + R_2^{(c)} C_2$$



$$R_1^{(c)} = R_1 // \left[\frac{R_N + R_2}{1 + g_{mS} R_2} \right] \simeq \frac{1}{g_{mS}}$$

$$R_2^{(c)} = R_2 // \left[\frac{R_N + R_1}{1 + g_{mS} R_1} \right] \simeq \frac{1}{g_{mS}}$$

so

$$p_2 \simeq - \frac{1}{(C_1 + C_2)\frac{1}{g_{mS}}} = - \frac{g_{mS}}{C_1 + C_2} \quad \text{(UNCHANGED)}$$

Therefore if we place the second pole at the GBWP:

$$\frac{C_T}{g_{m1}} = \frac{C_1 + C_2}{g_{mS}} \implies C_T = (C_1 + C_2)\frac{g_{m1}}{g_{mS}}$$

then

$$\left[R_N - \frac{1}{g_{mS}} \right] \cancel{sC_T} = \frac{\cancel{C_T}}{g_{m1}} \implies R_N = \frac{\cancel{C_T}}{\cancel{C_T}}\left(\frac{1}{g_{m1}} + \frac{1}{g_{mS}} \right) = \frac{2}{g_{mS}} \quad (\text{if } g_{mS} = g_{m1})$$

We may implement the resistor by using a MOSFET in ohmic region

$$I_{DS} = \frac{1}{2} \mu_n C'_{ox} \frac{W}{L} \left[(V_{GS} - V_T) V_{DS} \cdot 2 - V_{DS}^2 \right]$$

$$g_o = \left. \frac{\partial I_{DS}}{\partial V_{DS}} \right|_{V_{DS} = 0} = \frac{1}{2} \mu_n C'_{ox} \frac{W}{L} (2 V_{ov})$$

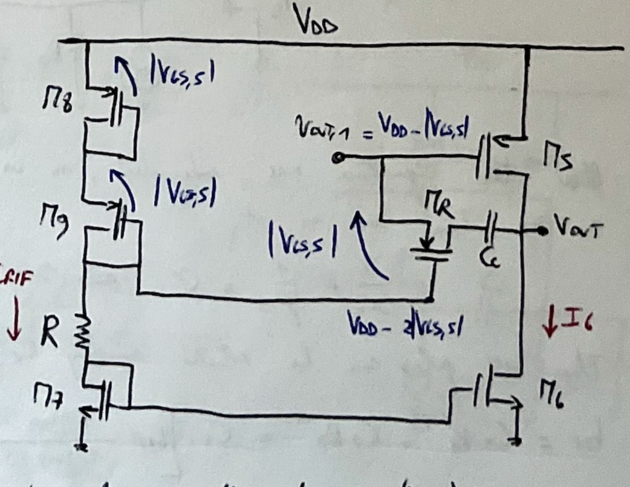
[Just by chance is equal to the TRANSCONDUCTANCE of a MOSFET in saturation]

therefore we need

$$\frac{g_{mS}}{g_o} = 2 \implies \frac{(W/L)_S}{(W/L)_R} \frac{V_{ov,S}}{V_{ov,R}} = 2$$

If we set carefully $V_{ov,S} = V_{ov,R}$, then the accuracy depends on a ratio, which is optimal in integrated design. In order to achieve this result we need to <u>REPLICATE</u> the bias condition of M_S:

M_S and M_6 are sized in order to carry the same current with $V_{OUT} = \frac{V_{DD}}{2}$. This current is set by the reference branch of M_7, which can be set to be $\frac{1}{M}$ of I_6 by sizing the $\left(\frac{W}{L}\right)_7 = \frac{1}{M}\left(\frac{W}{L}\right)_6$. If now we choose $\left(\frac{W}{L}\right)_8 = \left(\frac{W}{L}\right)_9 = \left(\frac{W}{L}\right)_7$, then $|V_{GS,8}| = |V_{GS,9}| = |V_{GS,S}|$



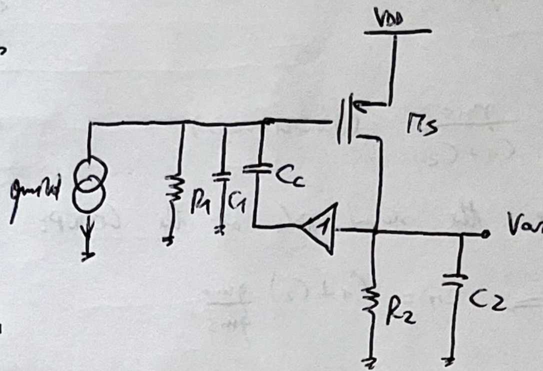
Therefore the position of the zero depends only on the ratio of $\frac{(W/L)_S}{(W/L)_R}$.

- **Voltage Buffer**

This solution preserves the Miller effect, but nulls the feed-forward path, so no zero!

→ Ideal

The three capacitors are dependent, so we expect just two poles. The dominant pole is still due to the Miller effect

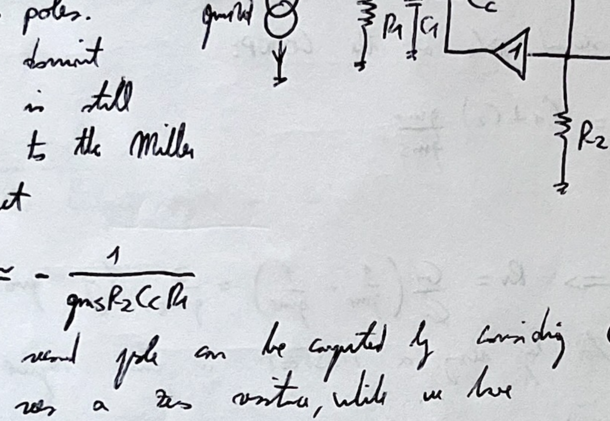


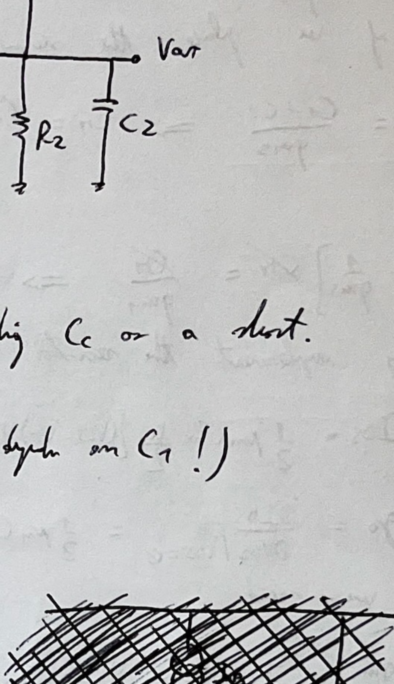
$$P_L \simeq -\frac{1}{g_{mS} R_2 C_C R_1}$$

The second pole can be computed by considering C_C as a short. C_1 sees a zero resistance, while we have

$$P_H \simeq -\frac{1}{C_2 \cdot \frac{1}{g_{mS}}}$$ (higher, it doesn't depend on C_1 !)

→ Real





Now the capacitors are independent, so we have an additional HF pole and a zero. The zero is at

$$\frac{1}{sC_C} + \frac{1}{g_{mB}} = 0 \Rightarrow z = -\frac{1}{C_C \cdot \frac{1}{g_{mB}}}$$ (output resist. of 1st stage toward ground)

The HF poles can be estimated considering C_C as a short

$$b_1 = C_1 R_1^{(C)} + C_2 R_2^{(C)} = C_1 \cdot \left[R_1 // \frac{1/g_{mB}}{1+g_{mS}R_2}\right] + C_2 \cdot \left[\frac{R_2}{1+g_{mS}R_2}\right] \simeq \frac{C_2}{g_{mS}}$$

$$b_2 = C_1 C_2 R_1^{(\infty)} R_2^{(1)(\infty)} = C_1 C_2 \frac{R_2}{g_{mB} g_{mS} R_2} = \frac{C_1 C_2}{g_{mB} g_{mS}}$$

So we get

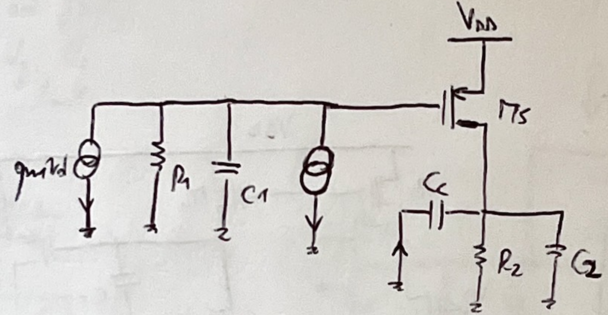
$$\left[s^2 \frac{C_1 C_2}{g_{mB} g_{mS}} + s \frac{C_2}{g_{mS}} + 1\right] = 0$$

$\downarrow$

$$\omega_0 = \sqrt{\frac{g_{mB} g_{mS}}{C_1 C_2}} \qquad Q = \frac{g_{mS}}{C_2} \sqrt{\frac{C_1 C_2}{g_{mB} g_{mS}}} = \sqrt{\frac{g_{mS} C_1}{g_{mB} C_2}} \qquad \text{(POLE PAIR)}$$

- AMUSA

$\longrightarrow$ Ideal



The Miller effect is still present ($v_c = g_{mS} v_S R_2$, $i_c = s C_c g_{mS} R_2 v_S$)
Same LF pole.
C_c and C_L are in //, so only two poles are expected. We can consider C_c as a short and compute the MF pole:

$$b_1 = C_2 \cdot 0 + C_1 \cdot \left(R_1 // \frac{1}{g_{mS}}\right) \simeq \frac{C_1}{g_{mS}}$$

$$p_H \simeq -\frac{1}{C_1 \cdot \frac{1}{g_{mS}}} \quad \text{(load capacitance plays no role!)}$$

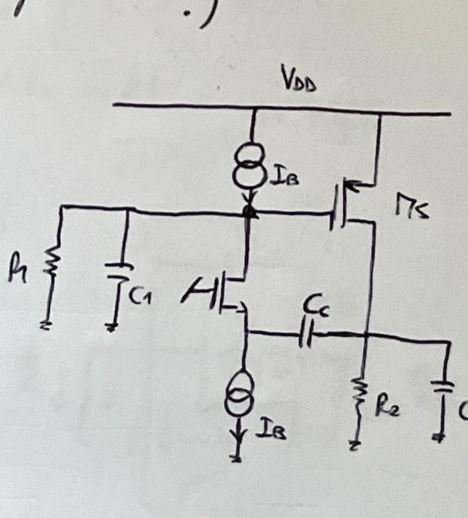
$\longrightarrow$ Real

We have a LHP zero at

$$\frac{1}{sC_c} + \frac{1}{g_{mB}} = 0$$

$\downarrow$

$$s = -\frac{1}{C_c \cdot \frac{1}{g_{mB}}} \quad \text{(output shorted to ground)}$$



and three poles. The LF pole is still due to the Miller effect, while we can estimate the MF ones by shorting C_c:

(27)

$$b_1 = C_1 \cdot \left[R_1 // \frac{1}{g_{m5}}\right] + C_2 \cdot R_2 // \left(\frac{1/g_{mB}}{1+g_{m5}R_1}\right) \simeq C_1 \frac{1}{g_{m5}}$$

$$b_2 = C_1 C_2 \cdot \frac{1}{g_{m5}} \cdot \frac{1}{g_{mB}}$$

So we get

$$\frac{C_1 C_2}{g_{m5} g_{mB}} s^2 + s \cdot \frac{C_1}{g_{m5}} + 1 = 0$$

$$w_0 = \sqrt{\frac{g_{m5}\, g_{mB}}{C_1 C_2}} \qquad Q = \frac{g_{m5}}{C_1} \cdot \sqrt{\frac{C_1 C_2}{g_{m5}\, g_{mB}}} = \sqrt{\frac{C_2\, g_{m5}}{C_1\, g_{mB}}}$$

For the same C_2, the Ahuja compensation has a higher Q factor than the voltage buffer compensation, so poles remain close and don't tend to split.

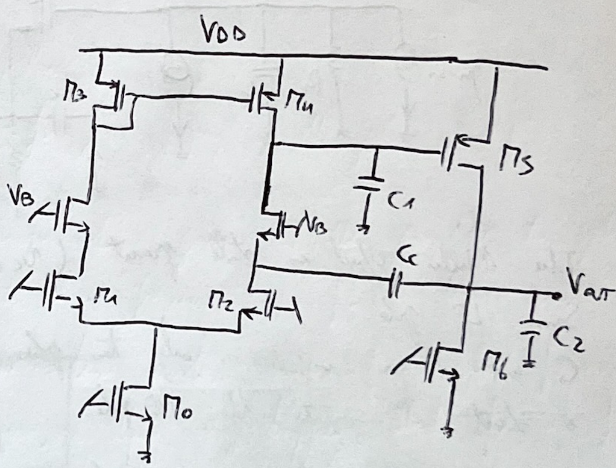
- **ALTERNATIVE AHUJA (CASCODE)**

Compute b_3, b_2, b_1 and a_2, a_1 and write

$$b_3 s^3 + b_2 s^2 + b_1 s + 1 = 0$$

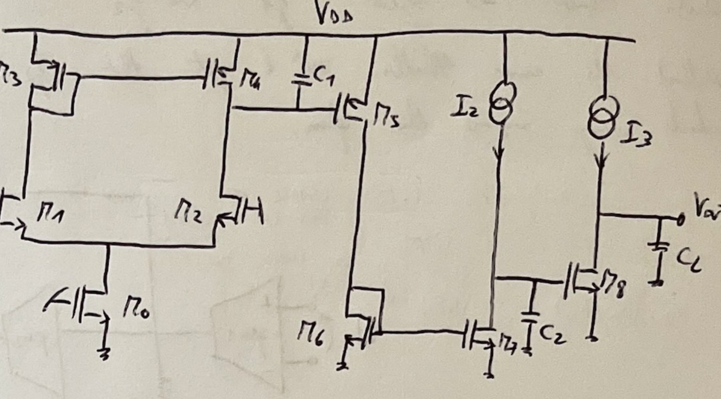
$$\Downarrow \simeq$$

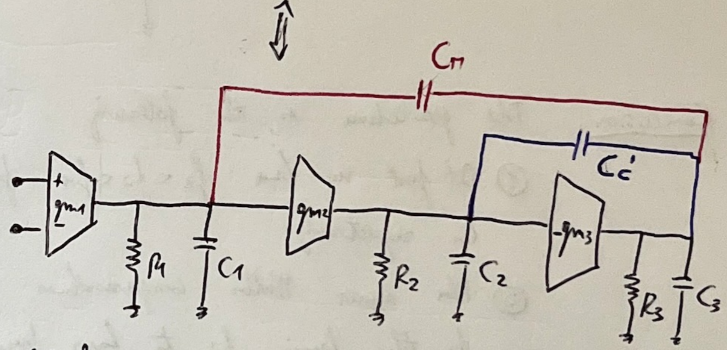
$$(1 + b_1 s)\left(1 + \frac{b_2}{b_1} s + \frac{b_3}{b_1} s^2\right) = 0$$



…MPENSATION (THREE-STAGE)

We use three-stage OTAs any time we need a large gain and do not have enough voltage headroom to accommodate cascodes:
now we have three high impedance nodes, so we need two sequential Miller compensations.





① Place a Miller capacitor across the third stage, in order to split f_2 and f_3. Let's consider only the cascade of the last two stages:

$$GBWP_{23} = \frac{g_{m2}}{2\pi C_c'}$$

$$f_3' = \frac{g_{m3}}{2\pi (C_3 + C_2)}$$

$$f_2 = \frac{g_{m3}}{2\pi C_c'} \quad \text{(POSITIVE)}$$

Usually $g_{m3} >> g_{m2}$, so we can neglect f_2.
Set φ_m of the 2-3 stage by placing $f_3' = 2\, GBWP_{23}$

② Place an outer Miller capacitor in order to further lower the frequency of the first pole

$$GBWP = \frac{g_{m1}}{2\pi C_M}$$

The second pole can be estimated by shorting C_M:

$$R_3^{(0)} = \frac{R_1 // R_3}{1 + g_{m2} R_2 g_{m3} (R_1 // R_3)} \quad \text{(seen by } C_1 \text{ and } C_3 \text{ in } //) \simeq \frac{1}{g_{m2} R_2 g_{m3}}$$

$$R_2^{(0)} = \frac{R_2}{1 + g_{m2} R_2 g_{m3} (R_1 // R_3)} \simeq \frac{1}{g_{m2} g_{m3} (R_1 // R_3)}$$

$$R_{C_c}^{(0)} = \frac{1}{g_{m2}} - \frac{1}{g_{m3}}$$

$$\downarrow$$

$$= \frac{g_{m3} - g_{m2}}{g_{m3}\, g_{m2}}$$

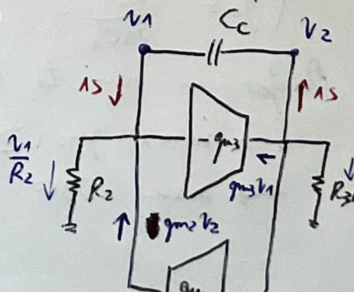
$$\begin{cases} i_s = -g_{m2} v_2 \\ i_s = -g_{m3} v_1 \end{cases}$$

$$\Downarrow$$

$$v_1 = -\frac{i_s}{g_{m3}}$$

$$v_2 = -\frac{i_s}{g_{m2}}$$

$$v_1 - v_2 = i_s \left(\frac{1}{g_{m2}} - \frac{1}{g_{m3}} \right)$$



(29)

A better estimate of the singularities can be evaluated by neglecting C_1 and C_3, via the capacitive loads at those nodes are dominated by the Miller capacitors.

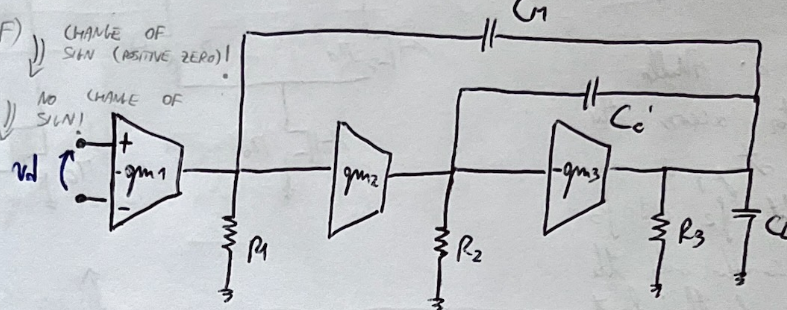
{ Inner Miller ⟹ pre-split f_2 and f_3 (ROOT LOCUS)
{ Outer Miller ⟹ split f_1 and f_2

Without the inner Miller we'd get that f_2 and f_3 may become a pole pair, which may degrade the φ_m.

$g_{m1}R_1\, g_{m2}R_2\, g_{m3}R_3$ (LF) ⇓ CHANGE OF SIGN (POSITIVE ZERO)!

$-g_{m1}R_3$ (MF) ⇓ NO CHANGE OF SIGN!

$-g_{m1}(R_2 /\!/ R_3)$ (HF)



CONCLUSION: The procedure is the following:

① At first we have $f_3 < f_2 < f_1$ frequency of the poles due to C_3, C_2 and C_1 respectively

② The inner Miller compensation gives us the possibility to split f_3 and f_2, thus bringing f_2 to lower frequencies and f_3 to higher ones

③ The outer Miller compensation instead moves f_1 to lower frequencies and f_2 to higher frequencies, so there may happen that f_2 and f_3 move closer to each other and become a pole pair.

Neglecting the fact that the outer Miller compensation brings f_2 and f_3 closer to each other, we have that

$$\begin{cases} GBWP \simeq \dfrac{g_{m1}}{2\pi C_M} \\ f_2 \simeq \dfrac{g_{m2}}{2\pi C_c'} \quad \left(\text{or } \dfrac{1}{2\pi}\,\dfrac{1}{C_c\left[\frac{g_{m3}-g_{m2}}{g_{m3}g_{m2}}\right]}\right) \\ f_3 \simeq \dfrac{g_{m3}}{2\pi C_3} \end{cases}$$

Therefore, in order to improve φ_m, we could either

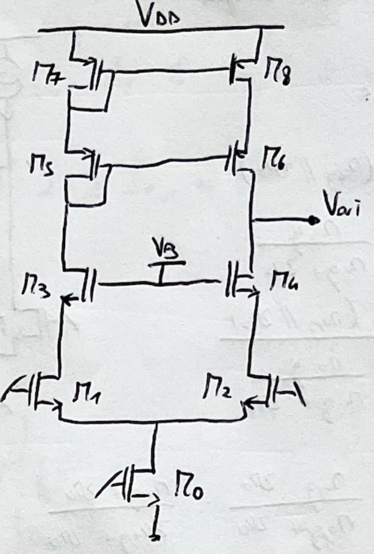
⟶ increase g_{m2} (MORE POWER DISSIPATION) and g_{m3}

⟶ decreasing GBWP by moving C_M (MORE SILICON AREA)

Single - Stage OTA

- **Telescopic Cascode**

This structure has a cascode both on the upper and lower side with respect to V_{OUT},
$\Rightarrow g_m R_{OUT} \simeq \frac{\mu^2}{2}$



[**DRAWBACK**: reduced voltage swing]

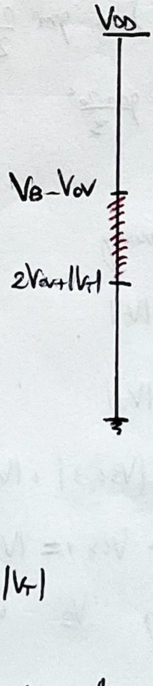
$\longrightarrow V_{OUT}|_{MAX} = V_{DD} - V_{SG,8} - V_{OV,6}$

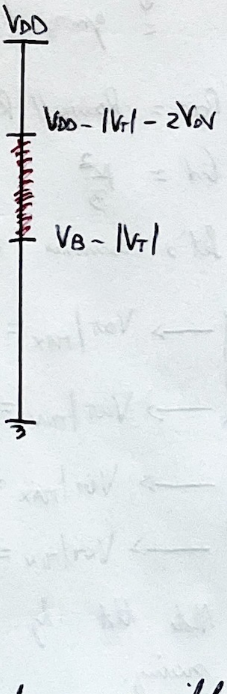
note that we may improve it by using an **ENHANCED MIRROR**

$\longrightarrow V_{OUT}|_{MIN} = V_B - |V_T|$

$\longrightarrow V_{CI}|_{MAX} = V_B - V_{GS,4} + |V_T| = V_B - V_{OV}$

$\longrightarrow V_{CI}|_{MIN} = V_{OV,0} + V_{GS,1} = 2V_{OV} + |V_T|$





Now, if we increase V_B we

INCREASE the common mode swing

DECREASE the output swing

and viceversa if we decrease V_B.

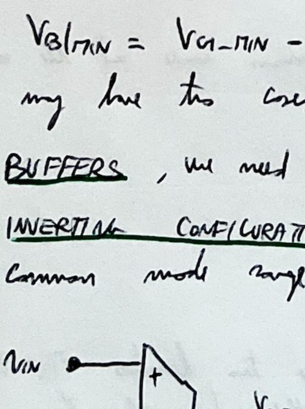
We have that

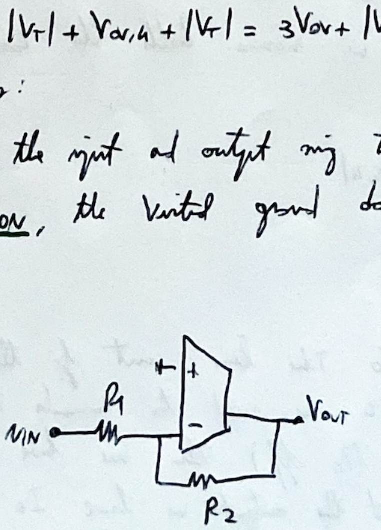
$$V_B|_{MAX} = V_{DD} - V_{SG,8} - V_{OV,6} = V_{DD} - 2V_{OV} - |V_T|$$

$$V_B|_{MIN} = V_{CI-MIN} - |V_T| + V_{OV,4} + |V_T| = 3V_{OV} + |V_T|$$

We may have two cases:

- **BUFFERS**, we need the input and output swing to overlap as much as possible
- **INVERTING CONFIGURATION**, the virtual ground doesn't pose requirements on the common mode range

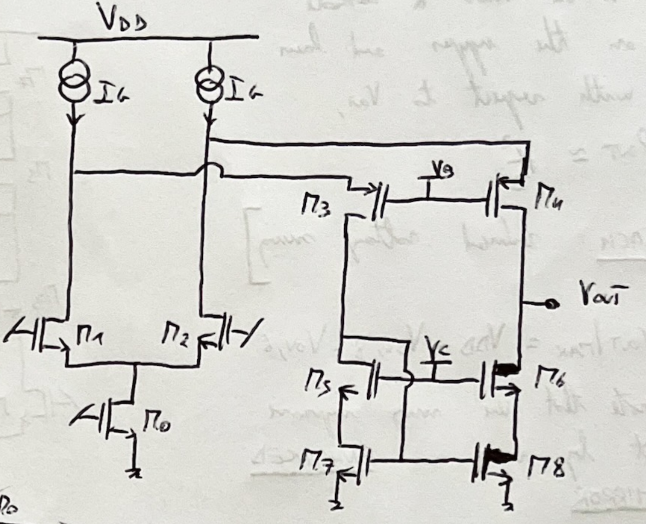




- **FOLDED CASCODE**

We avoid the series of many transistors in order to recover voltage swing

Let's compute the output resistance:



$$R_{DOWN} = g_m r_o^2$$

$$R_{up}^{(0)} \simeq g_m r_o (r_{o,g} \,//\, 2r_o)$$

$$G_{loop}(0) = -\frac{r_{o,g}}{r_{o,g} + 2r_o}$$

$$\Downarrow$$

$$R_{up} = \frac{g_m r_o (r_{o,g} \,//\, 2r_o)}{1 + \frac{r_{o,g}}{r_{o,g} + 2r_o}}$$

$$= g_m r_o \frac{r_{o,g} \cdot 2r_o}{\cancel{r_{o,g} + 2r_o}} \cdot \frac{\cancel{r_{o,g} + 2r_o}}{2r_{o,g} + 2r_o} =$$

$$= g_m r_o (r_{o,g} \,//\, r_o) \simeq g_m \frac{r_o^2}{2}$$

$$R_{OUT} = R_{DOWN} \,//\, R_{up} \simeq \frac{g_m r_o^2}{3}$$

$$G_d \simeq \frac{\mu^2}{3}$$

Let's consider the swing

$$\begin{cases} \longrightarrow V_{OUT}|_{MAX} = V_B + |V_T| \\ \longrightarrow V_{OUT}|_{MIN} = V_C - |V_T| \end{cases}$$

BIG DEAL!

$$\begin{cases} \longrightarrow V_{OI}|_{MAX} = V_B + |V_{SG,3}| + |V_T| \\ \longrightarrow V_{OI}|_{MIN} = V_{ov,0} + V_{GS,1} = |V_T| + 2V_{ov} \end{cases}$$

Note that by increasing V_B we increase both the common mode and the output swing.

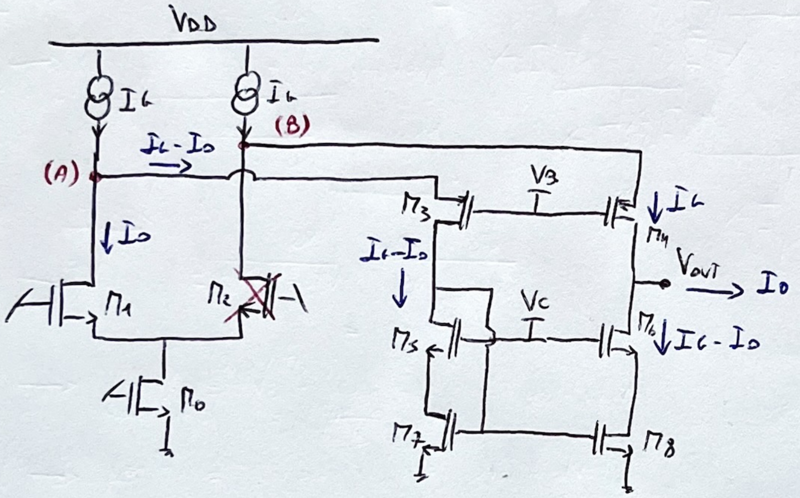
$$V_B|_{MAX} = V_{DD} - V_{ov,G} - |V_{GS,4}|$$

$$V_B|_{MIN} = V_{OUT,MIN} - |V_T|$$

- **SLEW RATE PERFORMANCE**

Let's consider I_0 current of M_0. The bias current of the two branches $M_3 - M_4$ will be $I_G - \frac{I_0}{2}$. To set I_G we need to consider that when the stage is fully unbalanced (for example M_2 off) then we have that the current through M_3 will be $I_G - I_0$ and at the output we have I_0.

If $I_G < I_0$, then M_3 and all his branch turns off, the potential of node (A) drops until when M_1 and M_0 go ohmic, thus matching $I_G = I_0'$. On the M_4 branch instead, I_G increases the potential of the output node, thus switching off M_4, or driving it into ohmic in order to match the needed output current, thus driving also I_G ohmic.
In this way nodes (A) and (B) reach voltages close to supply or ground, thus charging the stray capacitors and slowing down the circuit.
Therefore, not to degrade the performances, we need $I_G \geq I_0$, so more power dissipation.



Variability And Matching

CMRR and offset performances of a differential amplifier are heavily affected by components' tollerances. They depend on material properties variability and on dimensional parameters variability.

→ Resistors

$$R = \rho \frac{L}{W\Delta} = R_\square \cdot \frac{L}{W} \qquad R_\square \in [10\,\Omega/\square\,;\ 1\,k\Omega/\square]$$

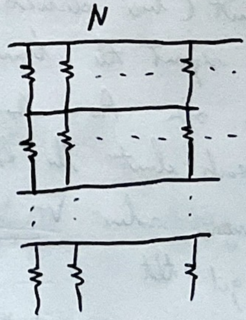
$$\Delta R = \frac{L}{W}\Delta R_\square + \frac{R_\square}{W}\Delta L + R_\square L \frac{\Delta W}{W^2}$$ (We sum them up in order to account for the worst case)

$$\downarrow$$

$$\frac{\Delta R}{R} = \frac{L}{W}\frac{\Delta R_\square}{R_\square} + \underbrace{\frac{R_\square}{W}\frac{\Delta L}{L} + R_\square \cdot L \frac{\Delta W}{W}}_{\text{Usually negligible}}$$

We assume that ρ and Δ are subjected to variability over a characteristic spatial length $\Lambda << W, L$. From this conclusion, we can divide the resistor in elementary volumes of height Δ and area $A_0 = \Lambda \cdot \Lambda$. Each elementary volume will have a normal distribution of $R_\square$ with $\overline{R_\square}$ mean value and $\sigma(\overline{R_\square})$ standard deviation.

We can see the resistor as a matrix of $m \times n$ resistors, $m = \frac{L}{\Lambda}$ and $n = \frac{W}{\Lambda}$, each of them being a sample of a Gaussian distribution.



① Rows $\quad G_1 = \sum_{i=1}^{N} G_{1,i}$

$$E[G_1] = \sum_{i=1}^{N} E[G_{1,i}] = N\,\overline{G_\square}$$

Now, being $G = \frac{1}{R}$ we may write

$$\frac{dG}{G} = \frac{dR}{R} \quad , \text{so} \quad \frac{\sigma^2(G)}{G^2} = \frac{\sigma^2(R)}{R^2}$$

assuming to deal with small variations, in order not to have PERCOLATIVE PATHS. (higher total resistance, higher variability)

$$\sigma^2(G_1) = \sum_{i=1}^{N} \sigma^2(G_{1,i}) = N\,\sigma^2(\overline{G_\square})$$

$$\frac{\sigma(G_1)}{G_1} = \frac{\sqrt{N}\,\sigma(\overline{G_\square})}{N\,\overline{G_\square}} = \frac{\sigma(\overline{G_\square})}{\overline{G_\square}} \cdot \frac{1}{\sqrt{N}} \simeq \frac{\sigma(\overline{R_\square})}{\overline{R_\square}} \cdot \frac{1}{\sqrt{N}} = \frac{\sigma(R_1)}{R_1}$$

② Column

$$R_{row} = \frac{1}{N\,\overline{G_\square}} = \frac{\overline{R_\square}}{N}$$

$$\frac{\sigma(R_{row})}{R_{row}} = \frac{\sigma(\overline{R_\square})}{\overline{R_\square}} \cdot \frac{1}{\sqrt{N}}$$

(34) So we get that the mean value of the total resistance is

$$\overline{R} = M \frac{\overline{R_\square}}{N}$$

and the standard deviation is

$$\frac{\sigma(\overline{R})}{\overline{R}} = \frac{\sqrt{M\,\sigma^2(R_{ROW})}}{M\frac{\overline{R_\square}}{N}} = = \frac{\sigma(R_{ROW})}{\sqrt{M}\,R_{ROW}} = \frac{\sigma(R_\square)}{R_\square}\cdot\frac{1}{\sqrt{MN}} = \frac{\sigma(R_\square)}{R_\square}\cdot\frac{\sqrt{A_0}}{\sqrt{WL}}$$

$$\Longrightarrow \quad \frac{\sigma(R)}{R} = \frac{K'_{\Delta R/R}}{\sqrt{WL}}$$

Let's consider now two nominally identical resistors R_1 and R_2: if they are manufactured close to each other, the mismatch is mainly due to statistical variability, so we can write

$$\Delta R = \sigma(\Delta R) = \sqrt{\sigma^2(R_1) + \sigma^2(R_2)}$$

$$\Downarrow$$

$$\frac{\sigma(\Delta R)}{\overline{R}} = \sqrt{\frac{\sigma^2(R_1)}{R^2} + \frac{\sigma^2(R_2)}{R^2}} = \sqrt{\frac{2K'^2_{\Delta R/R}}{WL}} = \frac{\sqrt{2}\,K'_{\Delta R/R}}{\sqrt{WL}} = \frac{K_{\Delta R/R}}{\sqrt{WL}} \quad \checkmark$$

$\longrightarrow$ TRANSISTORS

As for resistors, also for transistors the variability due to threshold and conductivity coefficient (hence electrical parameters) are dominant with respect to dimensional parameters. Let's segment the transistor area into a matrix of $M \times N$ elementary transistors with area $A_0 = \lambda \times \lambda$ over which the variability affects the V_T value. For each element the local V_T is a sample extracted from a gaussian distribution of mean value $\overline{V_T}$ and standard deviation $\sigma(\overline{V_T})$

We get that

[The overall threshold is the average of all the average contributions of each elementary volume]

$$\overline{V_T} = E\left[E[V_{T,0}]\right] = \overline{V_T} = \frac{1}{M\cdot N}\sum_{i=1}^{N}\sum_{j=1}^{M} V_{T,0}$$

$$\sigma^2(\overline{V_T}) = E\left[\left(E[V_{T,0}]\right)^2\right] = \frac{1}{(M\cdot N)^2}\cdot\sigma^2\left(\sum_{i=1}^{m} V_{T,i}\right) = \frac{1}{(M\cdot N)^2}\cdot \cancel{M\cdot N}\cdot\sigma^2(V_{T,0})$$

$$\Downarrow$$

$$\sigma(\overline{V_T}) = \sqrt{\frac{A_0}{W\cdot L}\,\sigma^2(V_{T,0})} = \frac{K'_{\Delta V_T}}{\sqrt{W\cdot L}}$$

{ $V_{T,i} \triangleq$ threshold of an elementary volume
$V_{T,0} \triangleq$ average threshold of an elementary volume
$\sigma(V_{T,0}) \triangleq$ standard deviation of an elementary volume }

and, considering two nominally equal transistors,

$$\sigma(\Delta V_T) = \frac{\sqrt{2}\,K'_{\Delta V_T}}{\sqrt{WL}} = \frac{K_{\Delta V_T}}{\sqrt{WL}}$$

CMRR

We can write, in general,

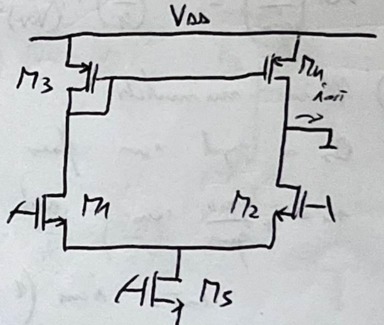
$$V_{OUT} = G_d v_d + G_{cm} v_{cm} = G_d \left[v_d + \frac{G_{cm}}{G_d} v_{cm} \right] = G_d \left[v_d + \frac{v_{cm}}{CMRR} \right]$$

so the common mode signal, together with the CMRR, is responsible for an input referred offset. Since CMRR depends on frequency, @ HF this offset contribution may be relevant.

Let's assume to have an input common mode signal at the input, it is almost entirely copied at this node, being thus follower:

$i_{cm} \simeq \frac{v_{cm}}{2 r_{o,g}}$ [common mode current flowing in the two branches]

Due to mismatches we may have that $i_{OUT} = \varepsilon\, i_{cm}$, so



$$G_{cm} = \frac{\varepsilon R_{OUT}}{2 r_{o,g}}$$

$$CMRR = \frac{g_{mD} R_{OUT}}{\frac{\varepsilon R_{OUT}}{2 r_{o,g}}} = \frac{2 g_{mD} r_{o,g}}{\varepsilon}$$

⟶ DETERMINISTIC ε

1. Mirroring error

   i_{cm} in the left-hand branch is partially lost into $r_{o,M}$ and not mirrored, thus causing

$$\varepsilon = i_{cm} \frac{1/g_{mM}}{\frac{1}{g_{mM}} + r_{o,M}} = i_{cm} \frac{1}{1 + g_{mM} r_{o,M}} \simeq \frac{i_{cm}}{g_{mM} r_{o,M}}$$

2. Unbalanced resistances at the input pair's drain nodes

$$R_{S,1} = \frac{\frac{1}{g_{mM}} + r_{o,D}}{1 + g_{mD} r_{oD}} =$$

$$R_{S,2} = \frac{\frac{r_{oD}}{g_{mD}}}{\frac{1}{g_{mD}} + r_{o,D}} = \frac{r_{o,D}}{1 + g_{mD} r_{oD}}$$

$$\varepsilon\, i_{cm} = 2 i_{cm} \frac{R_2}{R_1 + R_2} - 2 i_{cm} \frac{R_1}{R_1 + R_2} = \frac{R_2 - R_1}{R_2 + R_1} 2 i_{cm} = \frac{\frac{1}{g_{mM}}}{2 r_{o,D} + \frac{1}{g_{mM}}} 2 i_{cm}$$

$$R_2 - R_1 = \frac{-\frac{1}{g_{mM}}}{1 + g_{mD} r_{o,D}}$$

$$R_2 + R_1 = \frac{2 r_{o,D} + \frac{1}{g_{mM}}}{1 + g_{mD} r_{o,D}}$$

$$i_{cm}\,\mathcal{E} \simeq \frac{1}{2r_{o,D}\,g_{m,M}} \cdot 2 i_{cm} = \frac{i_{cm}}{r_{o,D}\cdot g_{m,M}}$$

$$\Rightarrow \left[\ \mathcal{E}_{DET} \simeq \frac{1}{r_{o,D}\,g_{m,M}} + \frac{1}{r_{o,M}\,g_{m,M}}\ \right]$$

→ STATISTICAL CMRR

$$dg_m = d(2kV_{ov}) = 2dk\cdot V_{ov} + 2k\,dV_T$$

$$\frac{dg_m}{g_m} = \frac{dk}{k} + \frac{dV_T}{V_{ov}}$$

Assuming that Δk and ΔV_T are statistically independent (they are not) we get

$$\sigma^2\left(\frac{\Delta g_m}{g_m}\right) = \sigma^2\left(\frac{\Delta k}{k}\right) + \frac{1}{(V_{ov})^2}\,\sigma^2(\Delta V_T)$$

[Because for example a variation of t_{ox} modifies C'_{ox}, which is present both in k and V_T formulas]

① Mirror mismatch

As a signal i_{cm} flows into M_3, $v_{g,3} = -\frac{i_{cm}}{g_{m3}}$, then we get

$i_4 = +\frac{g_{m4}}{g_{m3}}\,i_{cm}$ and so

$$i_{out} = i_{cm}\left(1 - \frac{g_{m4}}{g_{m3}}\right) = \frac{g_{m3}-g_{m4}}{g_{m3}}\,i_{cm} \simeq \frac{\Delta g_{m,M}}{g_{m,M}}\,i_{cm}$$

so

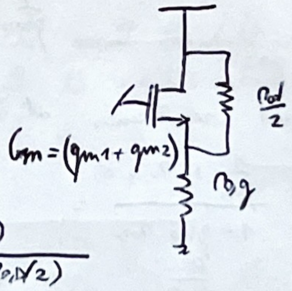
$$\mathcal{E}_M^{STAT} \simeq \frac{\Delta g_{m,M}}{g_{m,M}}$$

② Input mismatch

We need v_S, so we FOLD one transistor on the other

$$v_S = v_{cm}\cdot\frac{r_{o,g}\,//\,\frac{r_{o,D}}{2}}{r_{o,g}//\frac{r_{o,D}}{2} + \frac{1}{2g_m}}$$

$$= v_{cm}\cdot\frac{2g_{m,D}\,(r_{o,g}//r_{o,D}/2)}{1 + 2g_m\,(r_{o,g}//r_{o,D}/2)}$$



$$i_2 - i_1 = i_{out} = v_{cm}\,\frac{(g_{m1}-g_{m2})}{1+2g_m\,(r_{o,g}//r_{o,D}/2)}$$

$$\simeq \frac{\Delta g_{m,D}}{2g_{m,D}}\cdot\frac{r_{o,g}+\frac{r_{o,D}}{2}}{\frac{r_{o,g}\,r_{o,D}}{2}}\,v_{cm} = \frac{\Delta g_{m,D}}{g_{m,D}}\,\frac{v_{cm}}{2r_{o,g}}\left(\frac{2r_{o,g}+r_{o,D}}{r_{o,D}}\right)$$

$$\Rightarrow \mathcal{E}_D^{STAT} \simeq \frac{\Delta g_{m,D}}{g_{m,D}}\left(\frac{2r_{o,g}}{r_{o,D}} + 1\right)$$

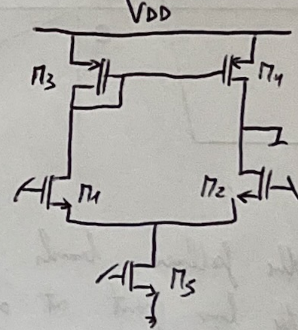
For $r_{o,g} \to +\infty$ we get CMRR $\rightarrow \dfrac{2g_{m,D}\,r_{o,g}}{\left(\frac{\Delta g_{m,D}}{g_{m,D}}\right)\frac{2r_{o,g}}{r_{o,D}}} = \dfrac{g_{m,D}\,r_{o,D}}{\left(\frac{\Delta g_{m,D}}{g_{m,D}}\right)}$

OFFSET

In a single-ended OTA, in principle, the output voltage should be at $\frac{V_{DD}-V_{EE}}{2}$ if no differential signal drives the two inputs. If the parameters involved deviate from nominal values, we may have that $V_{OUT} \neq \frac{V_{DD}-V_{EE}}{2}$, thus causing an output voltage offset. We usually describe the component as an ideal one with an offset equivalent generator placed between the input terminals, where

$$V_{OS} = \frac{V_{OUT,OS}}{L(0)}$$

Mismatches between transistors of the input stage are more critical, being thus amplified by the total gain of the stage.



→ **K MISMATCH M1 & M2**

$$I_1 = \left(k + \frac{\Delta k}{2}\right)(V_{GS}-V_T)^2 \quad ; \quad I_2 = \left(k - \frac{\Delta k}{2}\right)(V_{GS}-V_T)^2$$

$$|\Delta I| = |\Delta k (V_{GS}-V_T)^2| = |I_1 - I_2|$$

$$V_{OS}|_{\Delta k} = \frac{|\Delta I|}{g_m} = \frac{V_{ov}^2 \Delta k}{g_m} = \frac{V_{ov}^2 \cdot \Delta k}{2k V_{ov}} = \frac{\Delta k}{k}\cdot\left(\frac{V_{ov}}{2}\right)$$

→ **V_T MISMATCH M1 & M2**

$$I_1 = k\left(V_{GS} - V_T - \frac{\Delta V_T}{2}\right)^2 \quad ; \quad I_2 = k\left(V_{GS} - V_T + \frac{\Delta V_T}{2}\right)^2$$

$$|\Delta I| = \left| k\left[\cancel{(V_{GS}-V_T)^2} + \cancel{\frac{\Delta V_T^2}{4}} - 2V_{ov}\cdot\frac{\Delta V_T}{2} - \cancel{(V_{GS}-V_T)^2} + \left(-\cancel{\frac{\Delta V_T^2}{4}}\right) - 2V_{ov}\frac{\Delta V_T}{2}\right]\right| =$$

$$\downarrow$$

$$= 2k V_{ov}\cdot \Delta V_T$$

$$V_{OS}|_{\Delta V_T} = \frac{|\Delta I|}{g_m} = \frac{2k V_{ov}}{2k V_{ov}}\cdot \Delta V_T = \Delta V_T$$

→ **K MISMATCH M3 & M4**

$$I_3 = \left(k_M + \frac{\Delta k_M}{2}\right)(V_{GS}-V_T)^2 \quad ; \quad I_4 = \left(k_M - \frac{\Delta k_M}{2}\right)(V_{GS}-V_T)^2$$

$$|\Delta I| = \cancel{2}\,\frac{\Delta k_M}{\cancel{2}}(V_{GS}-V_T)^2$$

$$V_{OS}|_{\Delta k} = \frac{|\Delta I|}{g_m} = \frac{\Delta k_M}{2k_D V_{ovD}} V_{ovM}^2 = \frac{\Delta k_M V_{ov,M}^2}{2I} V_{ov,D} = \frac{V_{ov,D}}{2}\left(\frac{\Delta k_M}{k_M}\right)$$

→ **V_T MISMATCH M3 & M4**

$$I_3 = k_M\left(V_{GS} - V_T - \frac{\Delta V_T}{2}\right)^2 \quad ; \quad I_4 = k_M\left(V_{GS} - V_T + \frac{\Delta V_T}{2}\right)^2$$

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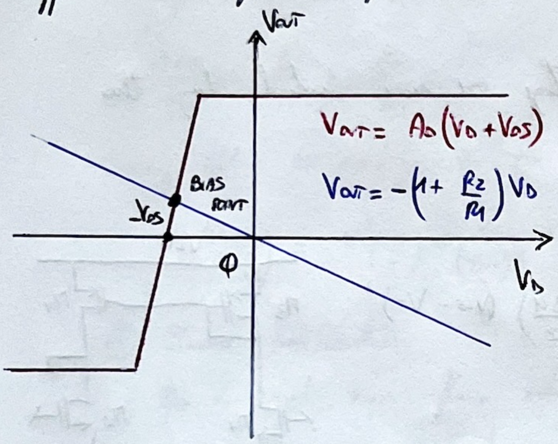
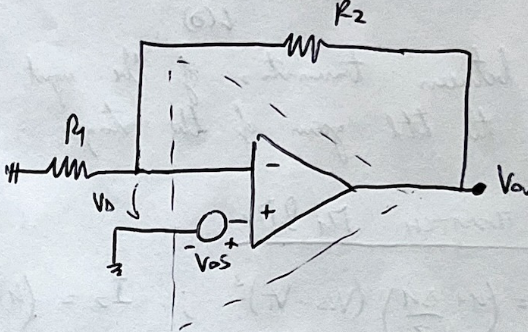
$$|\Delta I| = 2k_M V_{ov,M} \cdot \Delta V_T$$

$$V_{os}|_{\Delta V_T} = \frac{2k_M V_{ov,M}}{g_m} \Delta V_T = \frac{2k_M V_{ov,M}}{2I} \cdot V_{ov,D} \cdot \Delta V_T = \frac{2I}{V_{ov,M}} \cdot \frac{V_{ov,D}}{2I} \Delta V_T = \frac{V_{ov,D}}{V_{ov,M}} \Delta V_T$$

$$\Rightarrow \sigma^2(V_{os}) = \sigma^2(\Delta V_{T,D}) + \sigma^2(\Delta V_{T,M}) \left(\frac{V_{ov,D}}{V_{ov,M}}\right)^2 + \left[\sigma^2\left(\frac{\Delta k_D}{k_D}\right) + \sigma^2\left(\frac{\Delta k_M}{k_M}\right)\right] \left(\frac{V_{ov,D}}{2}\right)^2$$

Offset is compensated by **FEEDBACK**:





the feedback branch introduces a new relationship in the system, thus setting the bias point at a value lower than the power supply. In order to have this situation, it must be

$$\left(1 + \frac{R_2}{R_1}\right) << A_D$$

$$\downarrow$$

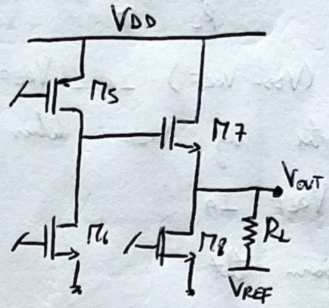
$$\underbrace{A_D \frac{R_1}{R_1 + R_2}}_{|G_{Loop}(0)|} >> 1$$

[Therefore in order to properly act on the feedback nodes, the loop gain must be properly sized as >> 1.]

Output Stages

→ **Class-A** (Transistors Are Always Active)

The simplest buffer stage is the source follower, where we set the output node at $\frac{V_{DD}-V_{EE}}{2}$ in order not to have a current into R_L when no input signal is applied $\left(V_{REF} = \frac{V_{DD}-V_{EE}}{2}\right)$



Let's assume to have a positive signal at the gate of M7. The gain of the stage is

$$\frac{V_{OUT}}{V_{G,7}} = \frac{R_L}{R_L + \frac{1}{g_{m7}}}$$

where g_{m7} is NOT constant, since it depends on the current value. As $V_{G,7}\uparrow$, the current $I_7\uparrow$, so $g_{m7}\uparrow$ and overall $\left(\frac{V_{OUT}}{V_{G,7}}\right)\uparrow$. On the negative swing instead, $I_7\downarrow$ and M7 may turn off. In order to avoid the output clipping we need to have

$$R_L \cdot I_8 \geq \Delta$$

being Δ the peak (negative) voltage at the output.

This stage is also adopted in POWER AMPLIFIERS, so we may compute the

POWER EFFICIENCY $\quad \eta = \frac{V_P^2/2R_L}{I_8 V_{DD}} \triangleq \frac{\text{POWER DELIVERED TO THE LOAD}}{\text{AVERAGE POWER DRAWN FROM SUPPLY}}$

Assuming that no clipping takes place, and that

$$\begin{cases} V_{DD} = 2V_{REF} \\ V_P \leq V_{REF} \end{cases}$$

$$\Rightarrow \eta \leq \frac{V_{REF}^2}{2R_L I_8 2V_{REF}} \leq \frac{V_{REF}^2}{4 \cdot V_{REF} \cdot V_{REF}} = \frac{1}{4}$$

→ **Class-B** (Push-Pull)

In order to avoid static power consumption, we may substitute the current generator with a pMOS follower, so that

- on the positive swing the nMOS turns on while the pMOS is off
- on the negative swing the pMOS turns on while the nMOS is off
- for $-V_T \leq V_{G,7} - V_{REF} \leq V_T$ we have a dead zone that causes CROSSOVER DISTORTION

therefore we have MORE DISTORTION, but BETTER POWER EFFICIENCY:

- The power delivered to the load is again $\frac{V_p^2}{2R_L}$ neglecting distortion
- On the positive swing a current flows from VDD to VREF, whose mean value is

$$I_A = \frac{1}{T/2}\int_0^{T/2} \frac{V_p}{R_L}\sin\left(\frac{2\pi}{T}t\right)dt = \frac{V_p}{R_L}\cdot\frac{2}{T}\frac{T}{2\pi}\cdot\left[\cos\left(\frac{2\pi}{T}t\right)\right]_{T/2}^{0}$$

$$\downarrow$$

$$= \frac{V_p}{R_L}\cdot\frac{1}{\pi}\,2 = \frac{2}{\pi}\frac{V_p}{R_L}$$

so $\bar{P} = (V_{DD} - V_{REF})\cdot I_A = V_{REF}\cdot I_A$

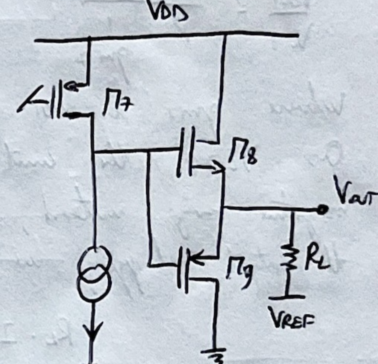
On the negative swing VREF delivers the same current towards ground, so again

$$\bar{P} = V_{REF}\cdot I_A.$$

Remembering again that

$$\begin{cases} V_p \le V_{REF} \\ V_{DD} = 2\,V_{REF} \\ \cancel{} \end{cases}$$

$$\eta \le \frac{\frac{V_{REF}^2}{2R_L}}{V_{REF}\cdot\frac{2}{\pi}\cdot\frac{V_{REF}}{R_L}} = \frac{\pi}{4} \simeq 78\,\%$$



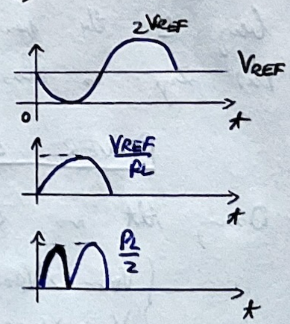
We have that across each transistor

$$\begin{cases} V_{DS}|_{max} = 2V_{REF} \\ I_{max} = \frac{V_{REF}}{R_L} \end{cases}$$ (neglecting the overdrive)

$$P(t) = \frac{V_{REF}}{R_L}\sin(\theta)\,V_{REF}\left[1 - \sin(\theta)\right]$$

$$\Downarrow$$

$$P_{MAX} = \frac{V_{REF}^2}{4R_L} \quad \left[\theta = \frac{\pi}{6}\right]$$ (half of the maximum power delivered to the load!)



⟶ **Class A-B**

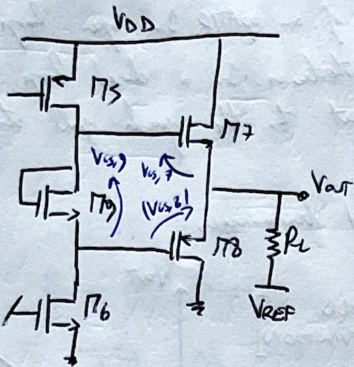
To limit distortion due to the dead-zone of the push-pull stage, we should bias the transistors M8 and M9 at the edge of their full conduction. We may implement this voltage shifter by using a transdiode:

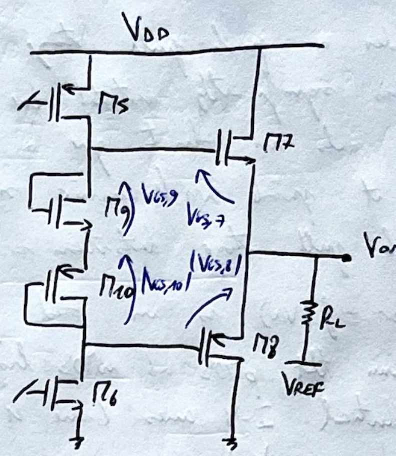
$$V_{GS,7} + V_{GS,8} = V_{GS,9}$$

{DEPENDENT ON ABSOLUTE VALUES (BAD)!}

$$\downarrow$$

$$\sqrt{\frac{I_7}{k_7}} + V_{T,n} + \sqrt{\frac{I_7}{k_8}} + V_{T,p} = \sqrt{\frac{I_6}{k_6}} + V_{T,n} \implies I_7 = \left(\frac{\sqrt{I_6/k_6} - V_{T,p}}{\sqrt{1/k_7} + \sqrt{1/k_8}}\right)^2$$





We may use the TRANSLINEAR PRINCIPLE by adding two complementary transdiodes, in such a way that the KVL in the M7 – M8 – M9 – M10 loop is depending only on VGS terms and we can control more carefully the currents

$$\sqrt{\frac{I_7}{k_7}} + \sqrt{\frac{I_7}{k_8}} + \cancel{V_{T,n}} + \cancel{V_{T,p}} = \sqrt{\frac{I_6}{k_9}} + \sqrt{\frac{I_6}{k_{10}}} + \cancel{V_{T,n}} + \cancel{V_{T,p}}$$

$$\downarrow$$

$$I_7 = I_6 \left(\frac{\sqrt{1/k_9} + \sqrt{1/k_{10}}}{\sqrt{1/k_7} + \sqrt{1/k_8}} \right)^2$$

So we can properly tailor the current (static one) in M7 and M8 by sizing the transistors M9 and M10.

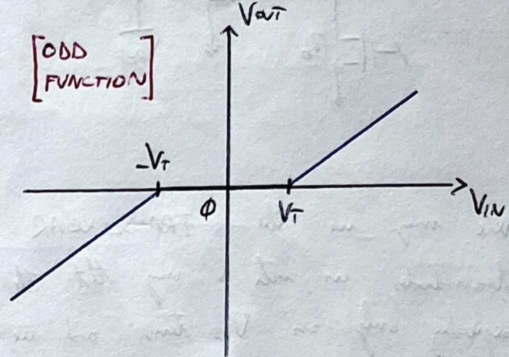
$$\begin{cases} \text{BIAS OF THE OUTPUT STAGE} \Rightarrow \text{set by } R_{L,MIN} \text{ (avoid clipping)} \\ \text{SIZING OF M9 AND M10} \Rightarrow \text{set by } I_7 \end{cases}$$

DISTORTION AND FEEDBACK

Let's consider a push-pull stage as output stage of an opAMP. In open-loop operation, the output is significantly distorted due to crossover distortion. In a closed loop configuration, the feedback acts in order to give a pre-distorted input to the push-pull stage and thus making it provide an harmonic signal.

In a push-pull stage distortion is dominated by odd harmonics:

→ $V_{OUT} = f(V_{IN})$ is an odd function, given that pMOS and nMOS have identical I-V curves (same k and same V_T)



→ if we drive the stage with

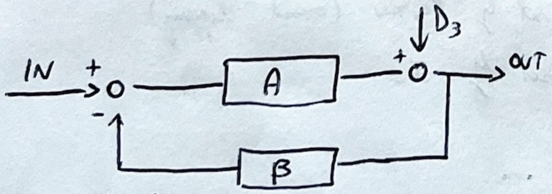
$v_{IN} = A_0 \sin(\omega t)$

the signal current through M_7 is a replica of the signal current through M_8 but with a $\frac{T}{2}$ shift, so $(T = \frac{2\pi}{\omega_1})$

$$i_1(t) = A_0 + A_1 \sin(\omega_1 t + \varphi_1) + A_2 \sin(\omega_2 t + \varphi_2) + A_3 \sin(\omega_3 t + \varphi_3) + \dots$$

$$i_2(t) = A_0 + A_1 \sin(\omega_1 t + \varphi_1 + \pi) + A_2 \sin(\omega_2 t + \varphi_2 + 2\pi) + A_3 \sin(\omega_3 t + \varphi_3 + 3\pi) + \dots$$

$$i_L = i_1 - i_2 = 2A_1 \sin(\omega_1 t + \varphi_1) + 2A_3 \sin(\omega_3 t + \varphi_3) + \dots$$

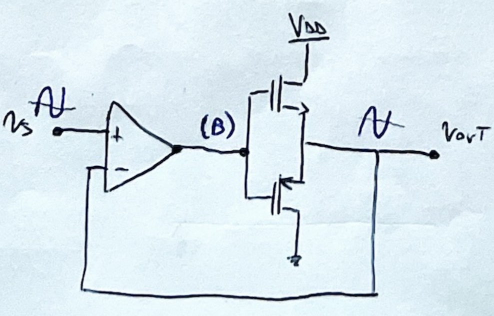


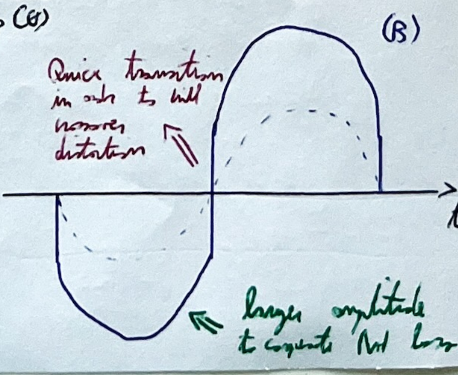
PRE - DISTORTION

$$-D_3^{OUT} \cdot A \cdot \beta + D_3 = D_3^{OUT}$$

$$\downarrow$$

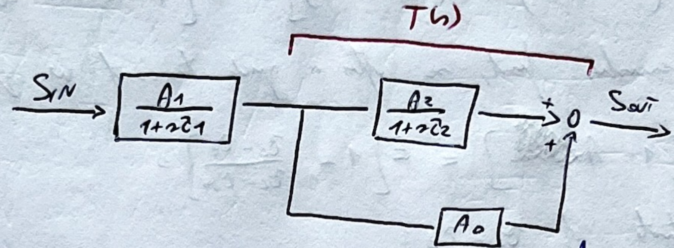
$$D_3^{OUT} = \frac{D_3}{1 + A\beta} \implies \frac{D_3^{OUT}}{D_3} = \frac{1}{1 - G_{loop}(0)}$$





In-Band Zero-Pole Doublets

In-band zero-pole doublets appear any time we need for example to drive a large load capacitance with a miller-compensated OTA stage, or when we use a FEED-FORWARD compensation technique:



→ low-pass block with broad band and low gain (GBWP trade-off)

$$T(s) = \frac{A_2}{1+s\tau_2} + A_0 = \frac{1 + s\left(\frac{A_0\tau_2}{A_2+A_0}\right)}{1+s\tau_2}(A_2+A_0)$$

so we get a zero @ $\tau_z = \frac{A_0}{A_2+A_0}\tau_2 < \tau_2$ (so $f_z > f_2$).

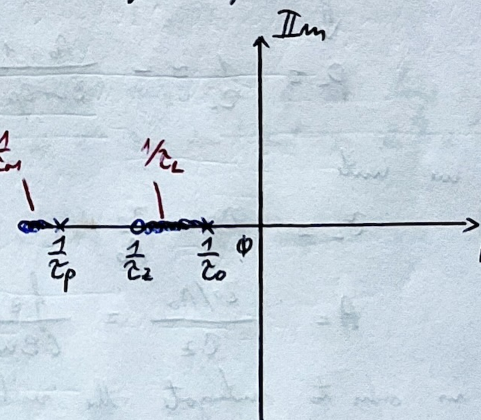
These doublets increase φ_m but worsen the transient response of the amplifier.

Let's consider

$$A(s) = \frac{A_0(1+s\tau_z)}{(1+s\tau_0)(1+s\tau_p)} \quad \text{(T.F. of a miller-compensated OTA)}$$

and $\tau_p < \tau_z < \tau_0$. The closed loop T.F. in a buffer configuration will be

$$H(s) = \frac{(1+s\tau_z)}{(1+s\tau_L)(1+s\tau_H)}$$



Let's consider the response to a step function E

$$V_{OUT}(s) = \frac{E}{s}H(s)$$

$$\downarrow$$

$$= \frac{E}{s}\,\frac{1+s\tau_z}{(1+s\tau_L)(1+s\tau_H)}$$

$$\downarrow$$

$$= \frac{E}{s}\left[\frac{A}{1+s\tau_L} + \frac{B}{1+s\tau_H}\right]$$

where

$$A = \lim_{s\to -\frac{1}{\tau_L}} H(s)(1+s\tau_L) = \frac{1+\tau_z\left(-\frac{1}{\tau_L}\right)}{1+\tau_H\left(-\frac{1}{\tau_L}\right)} = \frac{\tau_L-\tau_z}{\tau_L-\tau_H}$$

$$B = \lim_{s\to -\frac{1}{\tau_H}} H(s)(1+s\tau_H) = \frac{1+\tau_z\left(-\frac{1}{\tau_H}\right)}{1+\tau_L\left(-\frac{1}{\tau_H}\right)} = \frac{\tau_H-\tau_z}{\tau_H-\tau_L}$$

We get that

$$H(t) = \frac{A}{\tau_L} e^{-\frac{t}{\tau_L}} + \frac{B}{\tau_H} e^{-\frac{t}{\tau_H}}$$

so, by integration, we get:

$$v_{out}(t) = E\left[A\left(1-e^{-\frac{t}{\tau_L}}\right) + B\left(1-e^{-\frac{t}{\tau_H}}\right)\right] \overset{A+B}{=} E\left[1 - Ae^{-\frac{t}{\tau_L}} - Be^{-\frac{t}{\tau_H}}\right]$$

The fast exponential vanishes first, so after a quick transient we get

$$v_{out}(t) \simeq E\left[1 - A\right] = E\left[1 - \frac{\tau_L - \tau_Z}{\tau_L - \tau_H}\right] =$$

We need an estimate of $\tau_L - \tau_Z$:

$$G_{loop}(s) = -A_0 \frac{1+s\tau_z}{(1+s\tau_0)(1+s\tau_p)}$$

$$-G_{loop}(s) + 1 = 0$$

$$\Downarrow$$

$$-A_0 - A_0\tau_z \cdot s = 1 + s\tau_0 + s\tau_p + s^2\tau_0\tau_p$$

$$\downarrow$$

$$s^2\tau_0\tau_p + s(\tau_0 + \tau_p + A_0\tau_z) + A_0 + 1 = 0$$

$$\Downarrow$$

$$p_L = -\frac{1}{\tau_L} \simeq -\frac{A_0+1}{\tau_0+\tau_p+A_0\tau_z} \simeq -\frac{1}{\tau_z + \frac{\tau_0}{A_0}} \quad (\text{being } \tau_0 >> \tau_p)$$

So we write

$$\tau_L = \tau_z + \frac{\tau_0}{A_0}$$

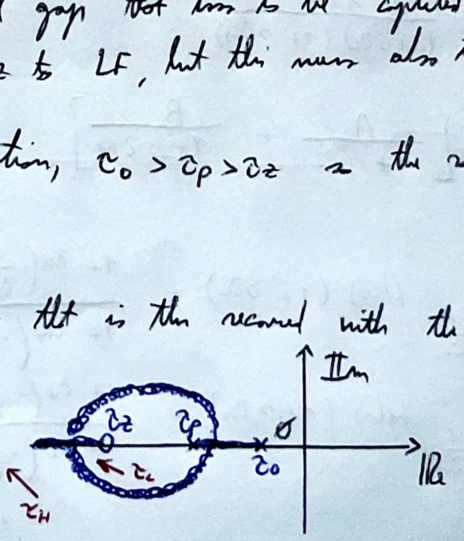
$$A = \frac{\tau_0/A_0}{\tau_z} = \frac{f_z}{GBWP}$$

so in order to mitigate the residual gap that has to be completed with the slow time constant, we must shift f_z to LF, but this means also to slow τ_L even down!

For the case of feed-forward compensation, $\tau_0 > \tau_p > \tau_z$ so the root locus is different and we expect

$$\tau_L < \tau_z$$

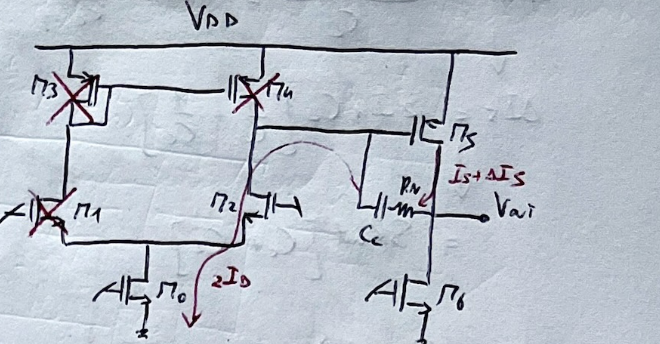
so the response has an overshoot that is then recovered with the slow time constant τ_L



Slew Rate

The slew-rate performance is related to the finite current available to charge capacitors.

If we consider a large signal driving for example M_2 it fully unbalances the differential couple, thus switching M_1-M_3-M_4 off and letting I_{M_0} flow into C_c. As a result, $V_{G,5} \downarrow$ and M_5 carries



$$I_5 + \underbrace{\Delta I_5}_{2I_D}$$

The initial response is a linear ramp with $SR = \frac{I_0}{C_c}$. If we assume to have the OTA in a buffer configuration, then we recover the exponential behavior when

$$\begin{cases} SR = \frac{\Delta V}{\tau} \\ \Delta V = V(\infty) - SR \cdot t_{SLEW} \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \Delta V = SR \cdot \tau \\ t_{SLEW} = \frac{V(\infty)}{SR} - \tau \end{cases} \Longrightarrow \quad v_{OUT}(t) = \begin{cases} SR \cdot t & (t \le t_{SLEW}) \\ V(\infty) - \Delta V \cdot e^{-\frac{(t - t_{SLEW})}{\tau}} & (t \ge t_{SLEW}) \end{cases}$$

This discussion is valid under the assumption that the rise time of the large input signal is $< \frac{\tau_L}{G(0)} = \frac{1}{GBWP(2\pi)}$ of the OTA, so that v_{OUT} doesn't react to the large signal at first.

If we consider an harmonic signal instead, whose amplitude is the maximum one allowed by the output dynamics, then

$$2\pi A_{MAX} f_{MAX} = \left.\frac{dv_{OUT}}{dt}\right|_{MAX} \le SR$$

$$\downarrow$$

$$f_{MAX} \le \frac{SR}{2\pi A_{MAX}} \triangleq \underline{\text{POWER BANDWIDTH}}$$

Finally, we can point out that

$$SR = \frac{I_0}{C_c} = \frac{V_{ov,1} \cdot g_{m1}}{C_c} = 2\pi V_{ov,1} \cdot GBWP$$

[Once GBWP is set, we need to increase $V_{ov,D}$ in order to increase SR, but this degrades E_{in}^2]

$\longrightarrow$ External SR

Let's consider now a load capacitance. We must distinguish now two cases

① large V_p (positive)

$M_1 - M_3 - M_4$ are off and $2I_1$ flows into C_c. Under the assumption that V_A reaches a steady state, the charge rate across C_c must be equal to the one across C_L, being it equal to $SR_{int} = \frac{2I_1}{C_c}$ imposed by the first stage, so it must be

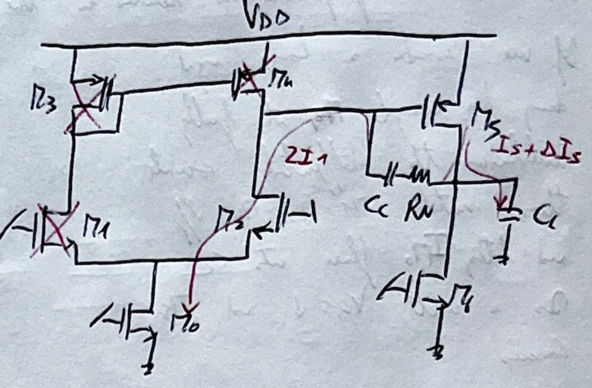
$$SR_{out}^{+} = \frac{\Delta I_5 - 2I_1}{C_L} = SR_{int} = \frac{2I_1}{C_c}$$

$\downarrow$

$$\Delta I_5 = 2I_1 C_L \left[\frac{1}{C_L} + \frac{1}{C_c}\right]$$

$\downarrow$

$$= SR_{int}\left[C_c + C_L\right]$$



② V_p large (negative)

M_2 off, $2I_1$ flows into $M_1 - M_3 - M_4$, M_5's current decreases

$$SR_{out}^{-} = \frac{\Delta I_5 - 2I_1}{C_L} = SR_{int} = \frac{2I_1}{C_L}$$ (if SR_{int} is more limiting than the output load)

$\downarrow$

$$\Delta I_5 = 2I_1 C_L \left[\frac{1}{C_L} + \frac{1}{C_c}\right] = SR_{int}(C_L + C_c)$$

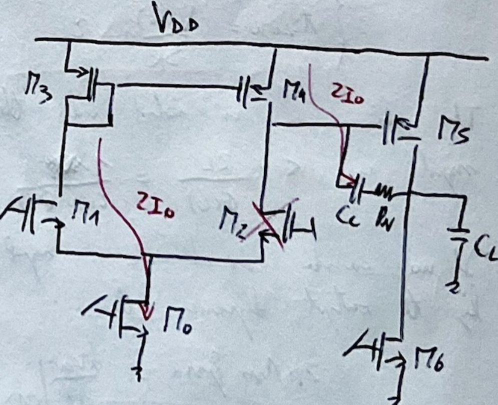
But if $\Delta I_5 > I_5$, then M_5 turns off and M_6 has to drive both C_c and C_L. In this case, at steady state the ramp rate across C_c matches the one across C_L, so

$$\frac{I_6 - I_4}{C_L} = \frac{I_4}{C_c} = SR_{out}^{-}$$

$\downarrow$

$$\begin{cases} I_4 = I_6 \cdot \dfrac{C_c}{C_L + C_c} < 2I_0 \\ SR_{out}^{-} = \dfrac{I_6}{C_L + C_c} \end{cases}$$

M_4 is ohmic! $\Uparrow$



In the end

$$SR_{out}^{-} = \min\left\{SR_{,int} \,;\, \frac{I_6}{C_L + C_C}\right\}$$

How to have that $SR_{,int}$ is the limiting one on both polarities?

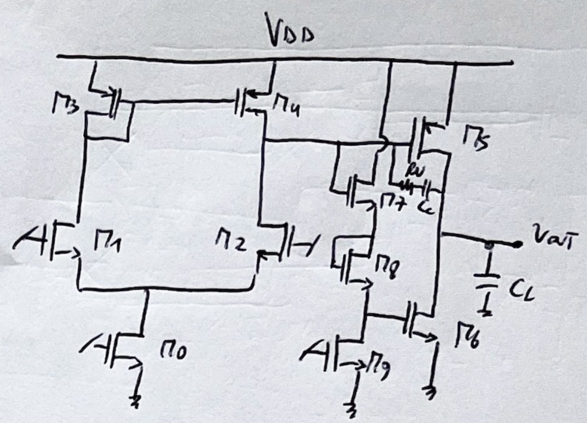
$\longrightarrow$ increase I_6

$\longrightarrow$ use a class A-B, raise M_6 current only when needed!

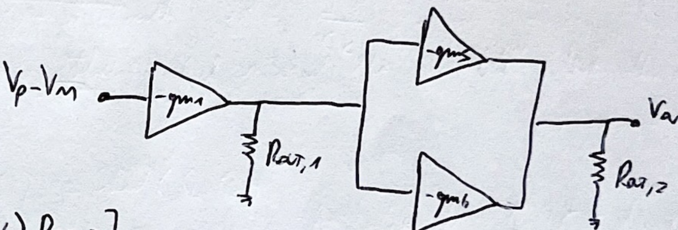
CLASS AB STAGE FOR SLEW RATE

M7 - M8 act as voltage shifters to properly set M6's bias. We size them in order to provide a shift of 0.8V + 0.8V.

In this way M6 starts in order to increase its current only when needed, so we do not increase static power dissipation!

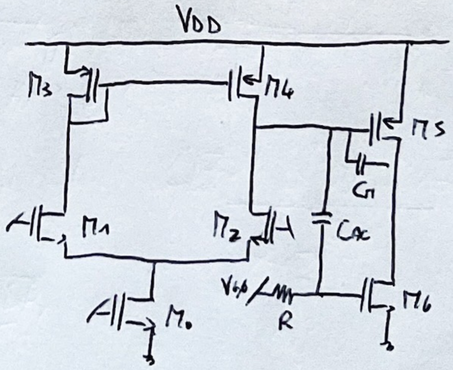


NOTE: the gain changes



$$G_d = \underbrace{\left[g_{m1} \cdot R_{OUT,1}\right]}_{G_1} \cdot \underbrace{\left[(g_{m5} + g_{m6})\, R_{OUT,2}\right]}_{G_2}$$

A possible alternative is the AC coupling: the capacitance C_{ac} makes it possible to properly bias the M6 transistor and also to let it come into action when needed.



FILTERS

→ Filter synthesis procedure:

1. FILTER MASK
2. FILTER TRANSFER FUNCTION
3. NETWORK IMPLEMENTATION (ideal)
4. NON-IDEALITIES OPTIMIZATION

→ Filters are divided (classified) into

LOW PASS ; HIGH PASS ; BAND PASS ; STOP-BAND ; EQUALIZERS (all pass)

The ideal filter should have

1) constant in band amplitude

2) linear phase shift proportional to w

⇓

DEM: Let's consider a signal falling in the pass-band of a filter. It is affected by a finite propagation delay τ, so

$$x(t) \longrightarrow \boxed{\text{FILTER}} \longrightarrow y(t) = A x(t-\tau)$$

τ must affect any harmonic component, so each harmonic suffers from a

$$\Delta\varphi = -\frac{2\pi}{T}\cdot\tau = -w\tau \quad \text{shift}$$

These conditions however cannot be fulfilled by a real filter, since

DEM: Let's consider an ideal LP on a bilateral frequency axis, so a rectangle. Its inverse Fourier transform is a

$$h(t) = \frac{1}{2\pi}\int_{-w_c}^{w_c} H_0 e^{jwt}\,dw = \frac{H_0}{2\pi j t}\left[e^{jwt}\right]_{-w_c}^{w_c} = \frac{H_0}{\pi t}\sin(w_c t)$$

$$\downarrow$$

$$= \frac{w_c H_0}{\pi}\,\frac{\sin(w_c t)}{w_c t} = h_0 \cdot \operatorname{sinc}(w_c t)$$

$h(t) \neq 0$ for $t < 0$ ⇒ ANTI-CAUSAL , NOT FEASIBLE !

→ Important definitions for the FILTER MASK are

A_{BP}	ATTENUATION IN BAND-PASS (maximum)	
A_{SB}	ATTENUATION IN STOP-BAND (minimum)	
w_{BP}	CUT-OFF FREQUENCY	
w_{SB}	LOWER LIMIT OF THE STOP-BAND	
$w_0 = \sqrt{w_{BP}^{+}\, w_{BP}^{-}}$	CENTRAL FREQUENCY OF A BAND-PASS FILTER	
$Q = \frac{w_0}{\Delta w}$	Q FACTOR OF A BAND-PASS FILTER	

$K = \frac{\omega_{BP}}{\omega_{SB}} < 1$ SECTIVITY INDEX (lower = worst)

$\varepsilon_{BP} = \sqrt{10^{\frac{A_{BP}}{10}} - 1}$ MAXIMUM ATTENUATION INDEX IN BAND-PASS

$\varepsilon_{SB} = \sqrt{10^{\frac{A_{SB}}{10}} - 1}$ MINIMUM ATTENUATION INDEX IN STOP-BAND

$K_\varepsilon = \frac{\varepsilon_{BP}}{\varepsilon_{SB}} < 1$ DISCRIMINATION INDEX (lower = better)

$\tau_{GD} = -\frac{d\varphi}{d\omega}$ GROUP DELAY (ideally constant)

⟶ ALL POLES FUNCTIONS

- BUTTERWORTH

They require maximum band-pass and stop-band flatness

$$\frac{d^n}{d\omega^n}|H(j\omega)|^2\Big|_{\omega=0} = 0 \quad \text{for } n = 1, 2, \dots, 2n-1$$

$$\frac{d^k}{d\omega^k}|H(j\omega)|^2\Big|_{\omega=\infty} = 0 \quad \text{for } k = 1, 2, \dots, 2n-1$$

For a reference $\Omega_0 = 1$ rad/s angular frequency, these conditions are fulfilled by

$$|H(j\omega)|^2 = \frac{1}{1 + \omega^{2n}}$$

All poles are placed on a circle of radius $\Omega_0 = 1$ rad/s on the Gauss plane, while

$$Q = \frac{1}{2\xi} = \frac{|p|}{2\,\mathrm{Re}[p]} = \frac{1}{2} \text{ if REAL}$$

Poles are equally spread one from the next on the circle by $\frac{\pi}{n}$ and the ones with higher Q factor are $\frac{\pi}{2n}$ far from the Im[ ] axis.

In order to achieve a lower A_{BP} than 3dB we may choose a different Ω_0, thus having

$$|H(j\omega)|^2 = \frac{1}{1 + \left(\frac{\omega}{\Omega_0}\right)^{2n}}$$

Let's fulfill the specs:

$$\frac{1}{1 + \left(\frac{\Omega_{BP}}{\Omega_0}\right)^{2n}} \geq \frac{1}{1 + \varepsilon_{BP}^2} \Rightarrow \Omega_0 \geq \frac{\Omega_{BP}}{\varepsilon_{BP}^{1/n}}$$

$$\frac{1}{1 + \left(\frac{\Omega_{SB}}{\Omega_0}\right)^{2n}} \leq \frac{1}{1 + \varepsilon_{SB}^2} \Rightarrow \Omega_0 \leq \frac{\Omega_{SB}}{\varepsilon_{SB}^{1/n}}$$

DEM: $\Omega_0 \geq \frac{\Omega_{BP}}{\varepsilon_{BP}^{1/m}} \wedge \Omega_0 \leq \frac{\Omega_{SB}}{\varepsilon_{SB}^{1/m}}$, by multiplying the first times for $\varepsilon_{SB}^{1/m}$ we get

$$\frac{\Omega_{BP}}{\varepsilon_{BP}^{1/m}} \cdot \varepsilon_{SB}^{1/m} \leq \Omega_0 \, \varepsilon_{SB}^{1/m} \leq \Omega_{SB}$$

$$\Downarrow$$

$$\Omega_{BP} \cdot k_\varepsilon^{-1/m} \leq \Omega_{SB}$$

$$\downarrow$$

$$k_\varepsilon^{-1/m} \leq k^{-1}$$

$$\downarrow$$

$$\frac{1}{m} \ln(k_\varepsilon^{-1}) \leq \ln(k^{-1})$$

$$\downarrow$$

$$m \geq \frac{\ln(k_\varepsilon^{-1})}{\ln(k^{-1})}$$

And $\frac{\Omega_{BP}}{\varepsilon_{BP}^{1/m}} \leq \Omega_0 \leq \frac{\Omega_{SB}}{\varepsilon_{SB}^{1/m}}$

- **BESSEL**

They require the maximum phase linearity, so

$$\left.\frac{d^n \tau_{GD}}{d\omega^n}\right|_{\omega=0} = 0 \quad \text{for } n = 1, 2, \dots, m$$

They have poles placed outside the Ω_0 characteristic frequency circle, in order to reach a better group delay, thus also a lower selectivity.
In order to select them, we use tables referred to $\Omega_0 = 1$ rad/s and we try to see if a specific order with a specific Ω_0 fulfills the specs.

- **CHEBYSHEV - I**

They require the maximum selectivity, but with an in-band ripple.

$$\max\left\{1 - |H(j\omega)|^2\right\}_{\omega \leq 1} \leq \varepsilon_{BP}^2 \quad \text{maximum ripple requirement}$$

$$\left.\frac{d^n |H(j\omega)|^2}{d\omega^n}\right|_{\omega \to \infty} = 0 \quad \text{for } n = 1, 2, \dots, 2m-1 \quad \text{stop-band flatness}$$

The DC value must be set to 1 for odd order T.F. and to $\frac{1}{\sqrt{1+\varepsilon_{BP}^2}}$ for even order T.F., in order to have $|H(j\omega)| \leq 1$ in band.

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USEFUL CHEBYSHEV FORMULAS

$$m \geq \frac{\operatorname{arcCh}(k\varepsilon^{-1})}{\operatorname{arcCh}(k^{-1})}$$

$$\Gamma = \left(\frac{1+\sqrt{1+\varepsilon_{BP}^2}}{\varepsilon_{BP}}\right)^{1/m}$$

Consider only the ones with $\mathrm{Re}[\ ] < 0$

$$s_k = -\sin\left[(2k-1)\frac{\pi}{2m}\right]\frac{\Gamma^2-1}{2\Gamma} + j\cos\left[(2k-1)\frac{\pi}{2m}\right]\frac{\Gamma^2+1}{2\Gamma}$$

(for $k = 1, \dots, 2m$)

m = # of half oscillations in band-pass

→ POLES AND ZEROES

- **CHEBYSHEV II**

They have imaginary conjugated zeroes in stop band in order to have a ripple there and kill HF disturbances.

$$\max\left\{1-|H(j\omega)|^2\right\}_{\omega\geq 1} \geq \varepsilon_{SB}^2 \quad \text{min attenuation in stop band} \left(\varepsilon = \frac{1}{\varepsilon_{SB}}\right)$$

$$\left.\frac{d^k}{d\omega^k}|H(j\omega)|^2\right|_{\omega=0} = 0 \quad \text{for } k = 1, 2, \dots, 2m-1$$

- **CAUER**

They are a constant ripple both in stop-band and in band-pass. Once k and the in-band ripple are set, the stop-band ripple is determined

- **GENERALIZED ELLIPTICAL**

They give an additional degree of freedom, so we can independently set k and both ripples, but we get a bad τ_{GD}.

→ ADDITIONAL DEMONSTRATION

It can be shown that for a Chebyshev-type-I filter it is

$$|H(j\Omega)|^2 = \frac{1}{1+\varepsilon^2 C_m^2(\Omega)}$$

$\varepsilon \triangleq$ RIPPLE FACTOR

C_m = Chebyshev polynomial

where

$$C_m(\Omega) = \begin{cases} \cos\left[m \arccos\left(\frac{\Omega}{\Omega_{BP}}\right)\right] & \left(\frac{\Omega}{\Omega_{BP}} \leq 1\right) \\ \mathrm{Ch}\left[m \operatorname{settCh}\left(\frac{\Omega}{\Omega_{BP}}\right)\right] & \left(\frac{\Omega}{\Omega_{BP}} \geq 1\right) \end{cases} \quad (=1 \text{ for } \Omega = \Omega_{BP})$$

so

$$|H(j\Omega_{BP})|^2 = \frac{1}{1+\varepsilon^2} \implies \varepsilon = \sqrt{10^{\frac{A_{BP}}{10}}-1} = \varepsilon_{BP}$$

In order to derive $m = \frac{\operatorname{settCh}(k\varepsilon^{-1})}{\operatorname{settCh}(k^{-1})}$ write $|H(j\Omega_{SB})|^2 = \frac{1}{1+\varepsilon_{SB}^2 C_m^2(\Omega_{SB})} \dots$

ORMALIZATION AND TRANSFORMATION

Filter synthesis is performed in a domain Ω referred to a low pass filter with $\Omega_{Bp} = 1$ rad/s. Therefore we need suitable transformations.

$\longrightarrow$ **LOW PASS**

$$\Omega = \frac{w}{w_{Bp}}$$ in order to get $\Omega_{Bp} = 1$ rad/s

Starting from the normalized Butterworth polynomials referred to $\Omega_0 = 1$ rad/s we need first to set

$$\hat{s} = \Omega_0 \, p$$

p normalized low-pass with $\Omega_0 = 1$ rad/s

$\hat{s}$ " " " $\Omega_0 \neq 1$ rad/s

Then, we get

$$s = w_{Bp} \cdot \hat{s}$$

s low-pass

$\longrightarrow$ **HIGH PASS**

$$\Omega = \frac{w_{Bp}}{w}$$ in order to have $\Omega_{Bp} = 1$ rad/s and to flip the axis

$$\hat{s} = \frac{w_{Bp}}{s}$$

$\longrightarrow$ **BAND - PASS**

DEM: let's consider this transformation

$$\hat{s} = \hat{p} + \frac{1}{\hat{p}} \qquad \begin{cases} \hat{s} = \Lambda + j\Omega \\ \hat{p} = \alpha + jw \end{cases}$$

$$\downarrow$$

$$\hat{p}^2 - \hat{s}\hat{p} + 1 = 0$$

$$\downarrow$$

$$\hat{p} = \frac{\hat{s} \pm \sqrt{\hat{s} - 4}}{2} \Rightarrow \alpha + jw = \frac{\Lambda + j\Omega \pm \sqrt{\Lambda^2 - \Omega^2 + 2j\Lambda\Omega - 4}}{2}$$

Now, the Im[] axis of $\hat{s}$ ($\Lambda = 0$) is mapped into

$$\alpha + jw = \frac{j\Omega \pm j\sqrt{\Omega^2 + 4}}{2} = j\frac{\Omega}{2} \pm j\sqrt{1 + \left(\frac{\Omega}{2}\right)^2}$$

$$\downarrow$$

so $\alpha = 0$ and the Im[] axis of $\hat{s}$ is mapped into two points on the Im[] axis of $\hat{p}$.

In particular,

$$\Omega = 0 \Rightarrow w_{1,2} = \pm 1$$

$$\Omega = 1 \Rightarrow w_{1,2} = +1{,}62 \,;\; -0{,}62$$

Being $\Omega = 1$ the cut-off of the low pass filter, this is mapped into Ω_{Bp}^{+} and Ω_{Bp}^{-} of the band pass, so it sets the BW to 1. So, to have a BW smaller than 1, we should apply the transformation to a LP with $\Omega_{Bp} = \frac{1}{Q}$. Finally, to shift the central frequency, we should expand $\hat{p}$, thus being $s = \hat{p} \cdot w_0$

$$\Downarrow$$

$$\hat{s} = Q\tilde{s} = Q\left[\hat{p} + \frac{1}{\hat{p}}\right] = Q\left[\frac{s}{w_0} + \frac{w_0}{s}\right] = Q\left[\frac{s^2 + w_0^2}{s w_0}\right]$$

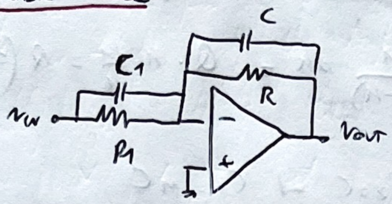
which in terms of radial frequency they correspond to

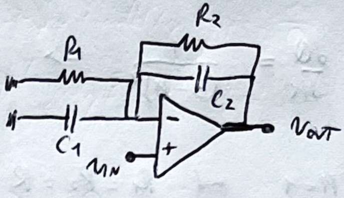
$$\Omega = Q\left(\frac{w^2 - w_0^2}{w w_0}\right)$$

Active Cells

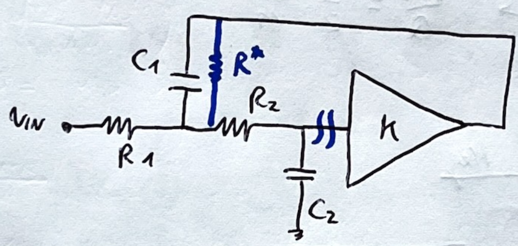
We can always split the T.F. in the product of 1^{st} and 2^{nd} order terms, so we can consider each of them as a block and then cascade all of them (carefully decoupled) in order to make the final T.F.. We can use OPAMPS!

⟶ **1^{ST} ORDER CELL**





⟶ **2^{ND} ORDER CELL (Sallen - Key)**



DEM:

$$G_{loop}(s) = +K \cdot \frac{R_1}{R_1 + R^*}$$

$$a_1 = R^* C_1$$

$$b_1 = C_2 (R_2 + R_1 /\!/ R^*) + C_1 \cdot (R^* /\!/ R_1)$$

$$b_2 = C_1 C_2 (R^* /\!/ R_1) R_2$$

$$G_{loop}(s) = +K \cdot \frac{R_1}{R_1 + R^*} \cdot \frac{1 + sC_1R^*}{1 + [C_2(R_2 + R_1/\!/R^*) + C_1(R^*/\!/R_1)]s + s^2 C_1 C_2 R_2 (R^*/\!/R_1)}$$

$$\Big\downarrow R^* \to +\infty$$

$$= +K \frac{sC_1R_1}{s^2 C_1 C_2 R_1 R_2 + s[C_1R_1 + C_2(R_1+R_2)] + 1}$$

By solving $G_{loop}(s) - 1 = 0$ we get the closed loop denominator:

$$s^2 (R_1R_2C_1C_2) + s[R_1(1-K)C_1 + (R_1+R_2)C_2] + 1 = 0$$

so $\omega_0 = \frac{1}{\sqrt{C_1C_2R_1R_2}}$ and $Q = \frac{\sqrt{R_1R_2C_1C_2}}{C_1R_1 + C_2(R_1+R_2) - KC_1R_1}$

$$\downarrow$$

$$= \frac{(R_1C_1)^{-1}\sqrt{R_1R_2C_1C_2}}{(1-K) + \frac{C_2}{R_1C_1}(R_1+R_2)} = \frac{\sqrt{\frac{R_2C_2}{R_1C_1}}}{(1-K) + \frac{C_2}{C_1}\left(1+\frac{R_2}{R_1}\right)}$$

55 Q depends on ratios ⇒ affected by on chip variations, mitigated by mixing

ω_0 depends on absolute values ⇒ affected by chip-to-chip variations

1 $K \neq 1$ and $R_1 = R_2 = R$ and $C_1 = C_2 = C$

$$\omega_0 = \frac{1}{RC} \qquad Q = \frac{1}{3-K} \implies K = 3 - \frac{1}{Q} \quad \text{dependent on } Q\,!$$

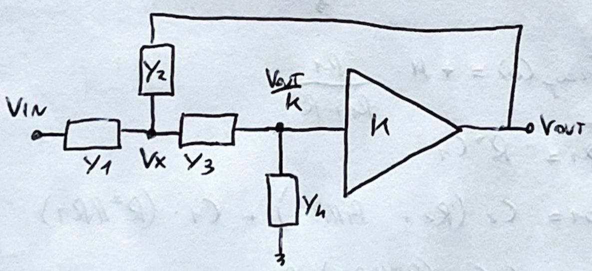
$$\frac{dQ}{dK} = \frac{1}{(3-K)^2} \implies \frac{dQ}{Q} = \frac{1}{(3-K)}\,dK = Q \cdot K \frac{dK}{K} = Q\left(3 - \frac{1}{Q}\right)\frac{dK}{K}$$

2 $K = 1$ and $R_1 = R_2 = R$ and $C_2 = C$ and $C_1 = mC$

$$\omega_0 = \frac{1}{\sqrt{m}\,RC} \qquad Q = \frac{\sqrt{m}}{2} \implies m = 4Q^2$$

$$\frac{dQ}{dm} = \frac{1}{4\sqrt{m}} \implies \frac{dQ}{Q} = \frac{1}{2}\frac{dm}{m}$$

→ <u>HIGH-PASS Sallen-Key</u>



<u>DEM</u>:

$$\begin{cases} (v_{IN} - V_x)\,Y_1 + (v_{OUT} - v_x)\,Y_2 = \left(v_x - \dfrac{v_{OUT}}{K}\right) Y_3 \\ v_x = \dfrac{v_{OUT}}{K}\left(1 + \dfrac{Y_4}{Y_3}\right) \end{cases}$$

↓

$$v_{OUT}\left(Y_2 + \frac{Y_3}{K}\right) - v_x\left[Y_1 + Y_2 + Y_3\right] = -v_{IN} Y_1$$

↓

$$v_{OUT}\left(Y_2 + \frac{Y_3}{K}\right) - \frac{v_{OUT}}{K}\left[1 + \frac{Y_4}{Y_3}\right]\left[Y_1 + Y_2 + Y_3\right] = -v_{IN} Y_1$$

↓

$$v_{OUT}\left[Y_2 + \frac{Y_3}{K} - \frac{Y_1}{K} - \frac{Y_2}{K} - \frac{Y_3}{K} - \frac{Y_1 Y_4}{K Y_3} - \frac{Y_2 Y_4}{K Y_3} - \frac{Y_4}{K}\right] = -v_{IN} Y_1$$

↓

$$\frac{v_{OUT}}{v_{IN}} = \frac{Y_1}{\frac{1}{K}\left[Y_1 + Y_2 + Y_3 + \frac{Y_1 Y_4}{Y_3} + \frac{Y_2 Y_4}{Y_3} + Y_4 - KY_2\right]} = \frac{K Y_1 Y_3}{Y_4\left[Y_1 + Y_2 + Y_3\right] + Y_1 Y_3 + Y_2 Y_3 (1 - K)}$$

→ UNIVERSAL CELL

Also called Kerwin - Huelsman - Newcomb (KHN)

DEM: let's consider a high-pass T.F.

$$\frac{v_{HP}}{v_{IN}}(s) = \frac{\gamma s^2}{\left(\omega_0^2 + \frac{s\,\omega_0}{Q} + s^2\right)}$$

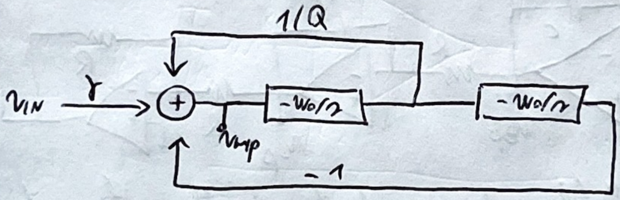
In order to let the integrators appear, let's divide for the highest power of s:

$$v_{HP}\left[1 + \frac{\omega_0}{Qs} + \frac{\omega_0^2}{s^2}\right] = \gamma\, v_{IN}$$

$$\downarrow$$

$$v_{HP} = \gamma\, v_{IN} - \frac{\omega_0}{s}\frac{v_{HP}}{Q} - \frac{\omega_0^2}{s^2} v_{HP}$$

BLOCK DIAGRAM:

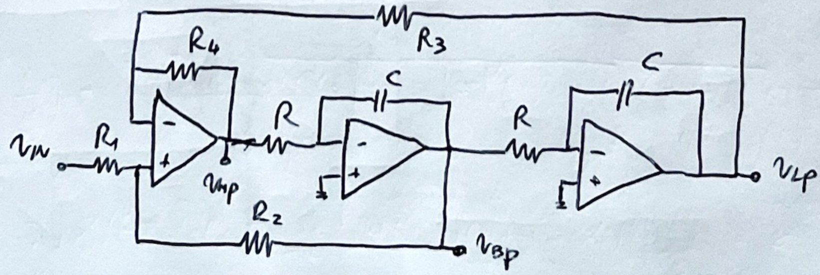


After the first integrator we get a BAND-PASS

$$\frac{v_{BP}(s)}{v_{IN}(s)} = -\frac{\gamma\,\omega_0\, s}{\left(s^2 + s\frac{\omega_0}{Q} + \omega_0^2\right)}$$

and after the second we get a LOW-PASS

$$\frac{v_{LP}(s)}{v_{IN}(s)} = \frac{\gamma\,\omega_0^2}{\left(s^2 + s\frac{\omega_0}{Q} + \omega_0^2\right)}$$



$$\Rightarrow v_{HP}(s) = \frac{R_2}{R_1+R_2}\cdot\frac{R_3+R_4}{R_3}\cdot v_{IN}(s) + \frac{R_1}{R_1+R_2}\cdot\frac{R_3+R_4}{R_3} v_{BP}(s) + \left(-\frac{R_4}{R_3}\right) v_{LP}(s)$$

So we get

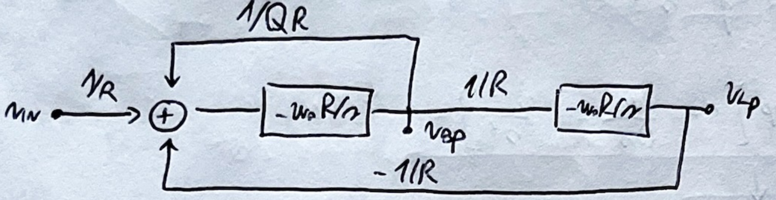
$R_4 = R_3$ ([redacted] -1) $\quad \gamma = \frac{2R_2}{R_1+R_2}$ and $\frac{1}{Q} = \frac{2R_1}{R_1+R_2}$

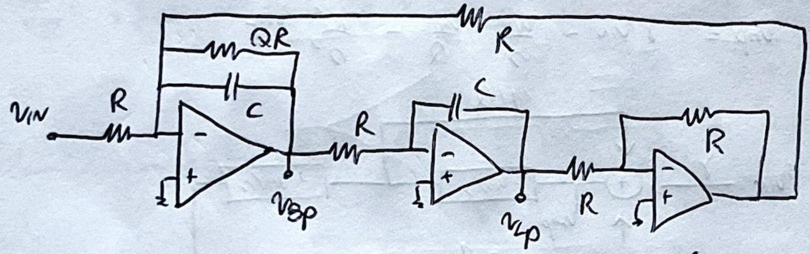
(57) $\frac{R_2}{R_1} = 2Q - 1$ and $\gamma = \left(2 - \frac{1}{Q}\right)$ dependent of each other!

If we combine the three outputs in a suited guard we may get a T.F. with also zeros

→ TOW-THOMAS AND FLEISCHER-THOMAS

We may use current signals instead of voltages by using an integrator V.C.. To this aim we need to divide ~~[illegible]~~ entering the integrator by $\frac{1}{R}$ and to multiply integrator T.F. by R.





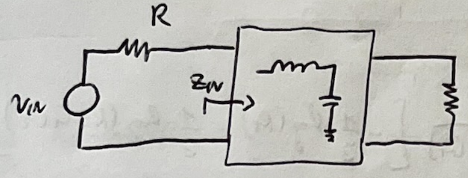
In a fully differential mode we don't need the inverter, it's enough to flip the cables.

By entering with v_{IN} in all three V.C. we get a Fleischer-Thomas cell with also two zeros.

LADDER NETWORKS

The idea is to exploit interactions between different components in the same cell in order to compensate sensitivities. In active cells instead, each cell contributes independently to sensitivity.

DEF: These networks are PASSIVE COMPONENTS, like this:



DOUBLY TERMINATED NETWORK

At the maximum of the T.F. $Z_{IN} = R$. At the nominal frequency ω the power delivered to the load can be written as

$$P_L = \frac{\frac{V_{OUT}^2}{2}}{R} = \frac{V_{IN}^2}{2R}\,|T(j\omega, X_0)|^2 \qquad X_0 \triangleq \text{nominal values for } L \text{ and } C$$

Let's assume to have a maximum @ $\omega = \omega^*$, then

$$\left.\frac{\partial P_L}{\partial \omega}\right|_{\omega=\omega^*} = \frac{V_{IN}^2}{2R}\cdot 2\,|T(j\omega^*, X_0)|\cdot \left.\frac{\partial}{\partial \omega}|T(j\omega, X_0)|\right|_{\omega=\omega^*} = 0$$

$$\downarrow$$

$$\left.\frac{\partial}{\partial \omega}|T(j\omega, X_0)|\right|_{\omega=\omega^*} = 0 = \left.\frac{\partial}{\partial X}|T(j\omega^*, X)|\right|_{X=X_0} \quad \text{being an absolute maximum}$$

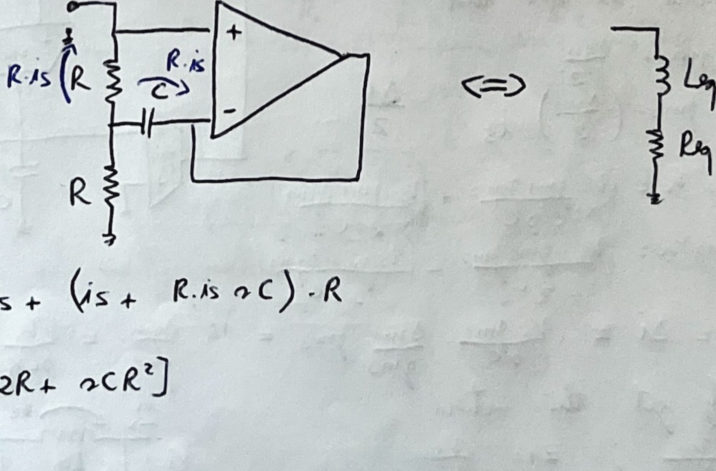
So the sensitivity of the T.F. to variations of L and C is nil at the passband and remains nil around it due to continuity (ORCHARD THEOREM)

How to implement inductors?

→ GYRATORS

They are active circuits to generate an inductive impedance

(1)



$$v_S = R\cdot i_S + (i_S + R\cdot i_S\, sC)\cdot R$$
$$\downarrow$$
$$= i_S\,[2R + sCR^2]$$

SENSITIVITY ANALYSIS

$$S_X^Y = \frac{\frac{\partial Y}{Y}}{\frac{\partial X}{X}} = \frac{\partial \log(Y)}{\partial \log(X)}$$

$\begin{cases} X = \text{circuit parameter} \\ Y = \text{filter parameter} \\ \log = \text{natural log } \ln \end{cases}$

EX: $w_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}}$

$$S_{R_1}^{w_0} = \frac{\partial \log(w_0)}{\partial \log(R_1)} = \frac{\partial}{\partial \log(R_1)}\left[-\frac{1}{2}\log(R_1) - \frac{1}{2}\log(R_2C_1C_2)\right] = -\frac{1}{2}$$

same for $S_{R_2}^{w_0}$ $S_{C_1}^{w_0}$ $S_{C_2}^{w_0}$, and for $S_K^{w_0} = 0$

EX: $Q = \frac{1}{[R_1(1-K)C_1 + (R_1+R_2)C_2]\,w_0}$

$$S_{R_1}^{Q} = \frac{\partial}{\partial \log(R_1)}\left[-\log(w_0) - \log\left(R_1(1-K)C_1 + (R_1+R_2)C_2\right)\right] =$$

$$= \frac{1}{2} - R_1\frac{\partial}{\partial R_1}\left[\log\left(R_1(1-K)C_1 + (R_1+R_2)C_2\right)\right] =$$

$$= \frac{1}{2} - R_1\frac{(1-K)C_1 + C_2}{R_1(1-K)C_1 + (R_1+R_2)C_2} =$$

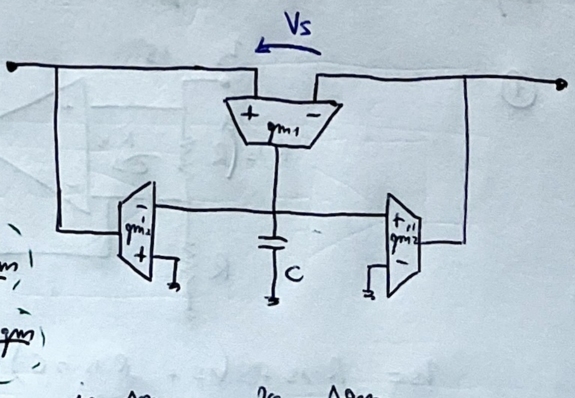
$$= \frac{R_1[1-K]C_1 + (R_1+R_2)C_2 - 2C_1R_1(1-K) - 2C_2R_1}{2[R_1(1-K)C_1 + (R_1+R_2)C_2]} =$$

$$= -\frac{C_1R_1(1-K) + (R_1-R_2)C_2}{2[R_1(1-K)C_1 + (R_1+R_2)C_2]}$$

The others are performed basically in the same way...

MISMATCH ISSUE IN GYRATORS

If the two g_{m2} transconductances do not match, we have that



$g_{m2}' = g_{m2} + \frac{\Delta g_m}{2}$

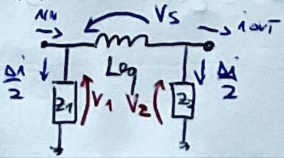
$g_{m2}'' = g_{m2} + \left(-\frac{\Delta g_m}{2}\right)$

$$i_{IN} = g_{m2}' v_C = \overbrace{v_s\frac{g_{m1}g_{m2}}{sC}}^{i_s} + \overbrace{v_s\frac{g_{m1}\Delta g_m}{2sC}}^{\frac{\Delta i}{2}}$$

$$i_{OUT} = g_{m2}'' v_C = v_s\frac{g_{m1}g_{m2}}{sC} - v_s\frac{g_{m1}\Delta g_m}{2sC}$$

$$i_{IN} - i_{OUT} = \Delta i = \frac{v_s g_{m1}}{sC}\cdot\frac{\Delta g_m}{g_{m2}}g_{m2} = i_s\frac{\Delta g_m}{g_{m2}} = \frac{v_s}{sL_{eq}}\frac{\Delta g_m}{g_{m2}}$$

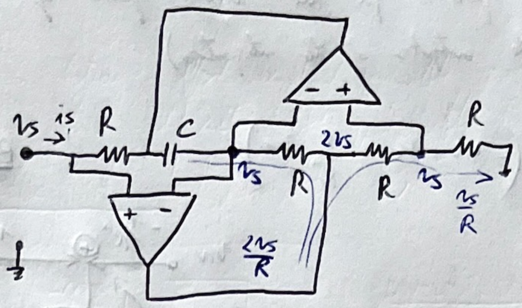
We basically get this network:



$z_2 = \frac{v_2}{\frac{\Delta i}{2}}$; $z_1 = \frac{v_1}{\frac{\Delta i}{2}}$

$\begin{cases} v_1 = \frac{v_s}{2} + v_{cm} \\ v_2 = -\frac{v_s}{2} + v_{cm} \end{cases}$

② Antoniou Circuit

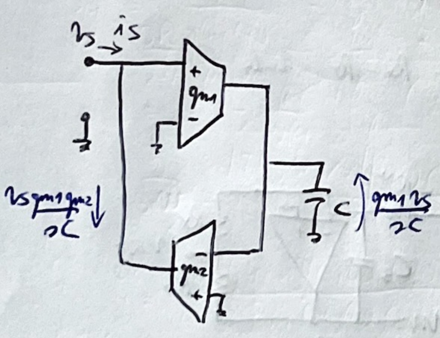


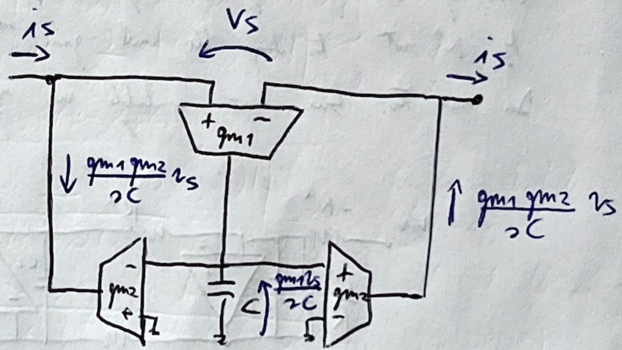
$$\frac{v_s}{R}\cdot\frac{1}{sC} = i_s \cdot R$$

↓

$$\frac{v_s}{i_s} = sCR^2$$

③ Other solutions

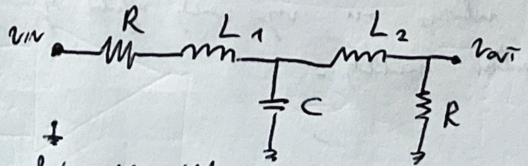




Problems of gyrators are
- limited GBWP of active elements
- noise

→ INTEGRATORS (DEF:)

Let's consider this 3rd order ladder network:



$$\begin{cases} \text{KVL}: & v_{IN} - v_C = (sL_1 + R)\, i_1 \\ \text{KVL}: & v_C - v_{OUT} = sL_2\, i_2 \\ \text{KCL}: & i_1 - i_2 = sC v_C \end{cases}$$

↓

find state variables (inductors currents, capacitor voltages)

Now write the currents as auxiliary voltages divided by an auxiliary resistance R'

$$\begin{cases} v_{IN} - v_C = \dfrac{sL_1 + R}{R'}\cdot v_1 \\ v_C - v_{OUT} = \dfrac{sL_2}{R'}\, v_2 \\ \dfrac{v_1 - v_2}{R'} = sC v_C \\ v_{OUT} = \dfrac{R}{R'}\, v_2 \end{cases}$$

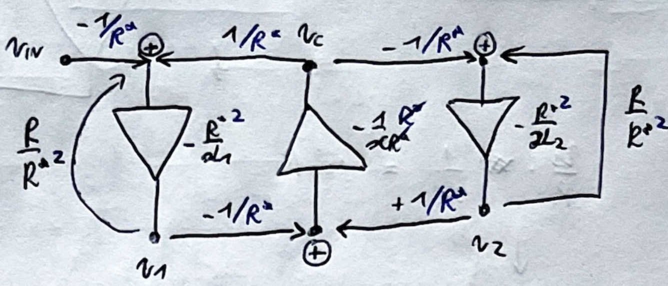
(61)

Now divide each equation for the highest power of s in order to let the integrators appear: (isolate the outputs of the integrators)

$\Downarrow$

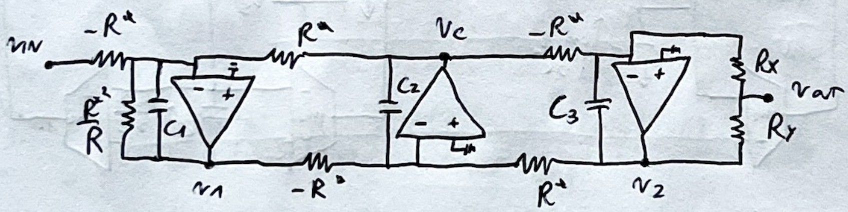
$$\begin{cases} v_1 = -\dfrac{R^*}{sL_1}\left(-v_{IN} + v_C + \dfrac{R}{R^*} v_1\right) \\ \text{[redacted]} \\ v_C = -\dfrac{1}{sCR^*}\left(-v_1 + v_2\right) \\ v_2 = -\dfrac{R^*}{sL_2}\left(-v_C + v_{OUT}\right) \\ v_{OUT} = \dfrac{R}{R^*} \cdot v_2 \end{cases}$$

BLOCK DIAGRAM



We may use virtual grounds in order to sum signals, so we need to divide each T.F. of the inputs of the integrator by $\frac{1}{R^*}$ and then to multiply each integrator T.F. by R^*, not to change the overall gain.

In order to have v_{OUT} available we need an R_X - R_Y divider

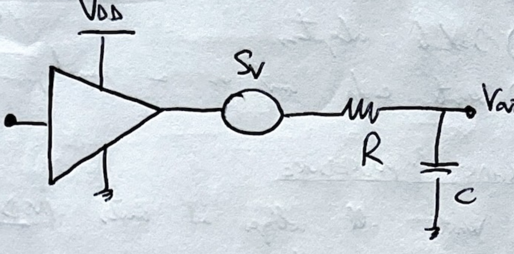


$$\begin{cases} R_X + R_Y = \dfrac{R^{*2}}{R} \\ v_{OUT} = \dfrac{R_X}{R_X + R_Y} v_2 = \dfrac{R}{R^*} v_2 \end{cases} \Rightarrow R_X = R^* \text{ and } R_Y = \frac{R^{*2}}{R} - R^*$$

These networks are very weakly affected by feedback capacitors' variability, but they are affected by resistor variability.

Dynamic Range $\triangleq$ ratio between maximum and minimum signal levels handled by the circuit.

DEM: Let's consider this circuit



The maximum output signal is $2V_{DD}$, so a sinusoid with $V_{PEAK} = \frac{2V_{DD}}{2}$, so

$$\langle V_{out}^2 \rangle = \frac{2^2 V_{DD}^2}{8}$$

while the RMS value of the noise due to just the resistor R is

$$4kTR \cdot \frac{1}{4RC} = \frac{kT}{C}$$

and the additional noise due to the amplifier is $F \cdot \frac{kT}{C}$, so we get

$$DR = \sqrt{\frac{2^2 V_{DD}^2/8}{(1+F)\,kT/C}} = 2V_{DD}\sqrt{\frac{C}{8kT(1+F)}}$$

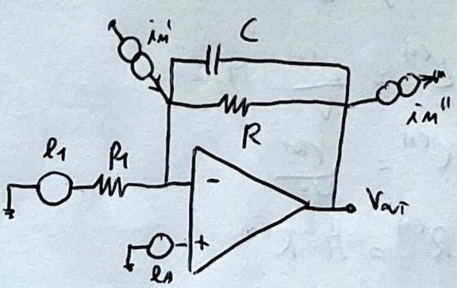
So, to increase the DR (which means to increase the number of bits of an A/D converter, being $DR = \frac{FSR}{LSB}$) we need to increase both C (more area) and V_{DD} (more power dissipation).

On each half-period the capacitance is charged up to $2V_{DD}$ and then discharged, so it dissipates

$$\underbrace{\frac{2V_{DD} \cdot C \cdot V_{DD}}{T}}_{\frac{\Delta Q}{\Delta t}}$$

→ **NOISE**

Let's consider this amplifier



① $e_{out,1} = e_1 \cdot \frac{R}{R_1} \frac{1}{1+sRC}$

⇓

$$S_{out,R_1} = S_{v,R_1} \left(\frac{R}{R_1}\right)^2 \frac{1}{4RC} = \frac{kT}{C} \frac{R}{R_1}$$

(2) $v_{out,2} \Rightarrow S_{I,R2} \cdot R^2 \cdot \frac{1}{4RC} = \frac{kT}{C}$

(3) $v_{out,A} \Rightarrow S_A \cdot \frac{\omega_M}{4} + S_A \left(1+\frac{R}{R_1}\right)^2 \frac{\omega_0}{4}$

we take into account also the finite GBWP of the amplifier, so that we consider the HF transfer to be limited by the ENBW $= \frac{\omega_M}{4}$

To be more accurate we should consider that the finite GBWP of the amplifier affects the overall T.F., so

$$e_{out,A} = e_A\left[1+\frac{R}{R_1}\frac{1}{1+sRC}\right]\cdot\frac{1}{\left(1+\frac{s}{\omega_M}\right)} = e_A\left(1+\frac{R}{R_1}\right)\frac{\left(1+\frac{s}{\omega_z}\right)}{\left(1+\frac{s}{\omega_0}\right)\left(1+\frac{s}{\omega_M}\right)}$$

ω_z being $= \frac{1}{C(R \| R_1)}$

If we write it in this way, we can use the tabulated integral

$$\int_0^{+\infty}\left|\frac{1+\frac{s}{\omega_z}}{\frac{s^2}{\omega_0\omega_M}+s\frac{(\omega_0+\omega_M)}{\omega_0\omega_M}+1}\right|^2 df = \frac{\omega_0\omega_M}{(\omega_0+\omega_M)}\frac{1}{4}\left[1+\frac{\omega_0\omega_M}{\omega_z^2}\right] = ENBW$$

so $e_{out,A} = S_A \cdot \left(1+\frac{R}{R_1}\right)^2 \cdot ENBW \xrightarrow{\omega_M \gg \omega_0} S_A \cdot \left(1+\frac{R}{R_1}\right)^2 \cdot \frac{\omega_0}{4}\left[1+\frac{\omega_0\omega_M}{\omega_z^2}\right]$

$\downarrow$

which corresponds to the approximated result. $= S_A\left(1+\frac{R}{R_1}\right)^2 \cdot \left[\frac{\omega_0}{4}+\frac{\omega_M}{4}\left(\frac{R_1}{R+R_1}\right)^2\right]$

(4) $S_{out} = \overbrace{S_{v\_in} G^2 \cdot \frac{\omega_0}{4}}^{INPUT} + \overbrace{4kTR \,\frac{\omega_0}{4}}^{R} + \overbrace{S_{v\_A} \cdot \left[\eta + (1+G)^2\right]\frac{\omega_0}{4}}^{AMPLIFIER} + \overbrace{4kTR\,G\,\frac{\omega_0}{4}}^{R1}$

$\downarrow$

$$= S_{v\_in} G^2 \frac{\omega_0}{4}\left[1+\frac{4kTR}{S_{v,IN}}\left(\frac{1+G}{G^2}\right)+\frac{S_{v\_A}}{S_{v\_IN}}\frac{\eta+(1+G)^2}{G^2}\right] = S_{v\_IN} \cdot G^2 \cdot \frac{\omega_0}{4}\left[1+F\right]$$

So, in order to minimize F, we need to

1) have $G > 1$
2) resistors whose noise is $< S_{v\_IN}$
3) amplifier whose noise is $< S_{v\_IN}$
4) limit the bandlimited noise by a suitable GBWP

$$\left[\begin{array}{l} G = \frac{R}{R_1} \\ \eta = \frac{\omega_M}{\omega_0} \end{array}\right]$$

Ladder Networks Summary

→ KL Procedure

1. Use KL in order to write relations among STATE VARIABLES (i_L, v_C)
2. Divide each member by the highest power of s
3. Isolate integrators
4. Write each current as an auxiliary voltage divided by R^*

→ Flowchart Procedure

1. Write all voltages and currents INSIDE COMPONENTS
2. Draw integrators
3. Use KL to derive the inputs of integrators (i_C, v_L)
4. Divide the branches going into V.C. by R^* and multiply the integrators T.F. for R^*
5. Multiply currents on the down side for R^*, divide branches going up for R^* and multiply branches going down for R^*
6. Multiply for (-1) inputs of integrators and integrators T.F.

→ Denormalization

Remember $\omega_0 \propto \frac{1}{\sqrt{LC}}$ and $Q \propto \omega_0 RC$, so to shift ω_0 to a value N times higher we need to set

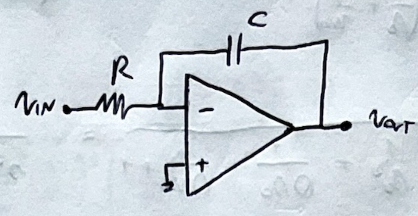
$$\begin{cases} L^{(1)} = \frac{L^{(0)}}{N} \\ C^{(1)} = \frac{C^{(0)}}{N} \end{cases}$$

This doesn't change Q.
Moreover, to further decrease C without changing ω_0 and Q we need to multiply L and R by M:

$$\begin{cases} L^{(1)} = \frac{L^{(0)}}{N} \cdot M \\ C^{(1)} = \frac{C^{(0)}}{MN} \\ R^{(1)} = M \cdot R^{(0)} \end{cases}$$

OP-AMP NON IDEALITIES

Let's consider an integrator: we have that the forward gain is the opAMP, while the $\beta(s)$ is



$$\beta(s) = \frac{R}{R + \frac{1}{sC}} = \frac{sCR}{1+sCR}$$

$$\begin{cases} CR = \tau_0 \\ \frac{1}{CR} = \omega_0 \end{cases}$$

$$\Downarrow$$

$$\frac{1}{\beta}(s) = \frac{1 + \frac{s}{\omega_0}}{\frac{s}{\omega_0}} = 1 + \frac{\omega_0}{s}$$

The Gloop(s) is equal to $G_{loop}(s) = -\frac{A_0}{(1+s\tau)}\frac{s\tau_0}{(1+s\tau_0)}$, it has a low frequency pole due to the opAMP, a zero in DC and a pole at ω_0, so the loop gain @ middle frequency is

$$|G_{loop}(MF)| = A_0\frac{\tau_0}{\tau} = \frac{GBWP}{f_0}$$ (we need $GBWP \geq 100 f_0$ to have a suitable loop gain)

and the closed loop transfer function has a pole at

$$\omega_L = \frac{\omega_0}{A_0} = \frac{1}{A_0RC}$$ (MILLER POLE)

and another pole at

$$\omega_H = GBWP$$

So, in the end,

$$H_{int}(s) = \frac{-A_0}{\left(1+\frac{s}{\omega_L}\right)\left(1+\frac{s}{\omega_H}\right)}$$

$$\begin{cases} \text{DC gain} \\ \text{LOW FREQUENCY POLE} \\ \text{HIGH FREQUENCY POLE} \end{cases}$$

$\longrightarrow$ DC GAIN AND LF POLE EFFECT

$$T(s) = \frac{\omega_0^2 \cdot \frac{1}{s^2}}{\left(s^2 + s\frac{\omega_0}{Q} + \omega_0^2\right)\cdot\frac{1}{s^2}} = \frac{\left(-\frac{\omega_0}{s}\right)^2}{\left(-\frac{\omega_0}{s}\right)^2 - \frac{1}{Q}\left(-\frac{\omega_0}{s}\right)+1} = \frac{H_{int}^2(s)}{H_{int}^2(s) - \frac{1}{Q}H_{int}(s)+1}$$

Let's consider now $H_{int}(s) = \frac{-A_0}{1+\frac{s}{\omega_L}}$ at LOW FREQUENCY, so we can write it as

$$H_{int}(s) = -\frac{1}{\left(\frac{1}{A_0} + \frac{s}{\omega_0}\right)}$$

and replace it into $T(s)$

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$$T(s) = \frac{\dfrac{1}{\left(\frac{1}{A_0}+\frac{s}{w_0}\right)^2}}{\dfrac{1}{\left(\frac{1}{A_0}+\frac{s}{w_0}\right)^2}+\dfrac{1}{Q\left(\frac{1}{A_0}+\frac{s}{w_0}\right)}+1} = \frac{1}{1+\frac{1}{Q}\left(\frac{1}{A_0}+\frac{s}{w_0}\right)+\left(\frac{1}{A_0}+\frac{s}{w_0}\right)^2}$$

$$= \frac{1}{\frac{s^2}{w_0^2}+\frac{2s}{w_0A_0}+\frac{1}{A_0^2}+\frac{1}{QA_0}+\frac{s}{Qw_0}+1} = \frac{w_0^2}{s^2+\frac{2sw_0}{A_0}+\frac{w_0^2}{A_0^2}+\frac{w_0^2}{QA_0}+w_0^2+\frac{sw_0}{Q}}$$

$$= \frac{w_0^2}{s^2+sw_0\left(\frac{1}{Q}+\frac{2}{A_0}\right)+w_0^2\left(1+\frac{1}{QA_0}+\frac{1}{A_0^2}\right)}$$

$$\Rightarrow w_0' = w_0\sqrt{1+\frac{1}{A_0Q}+\frac{1}{A_0^2}} \qquad \text{RADIAL FREQUENCY SHIFT}$$

$$\Rightarrow \frac{1}{Q'} \simeq \left(\frac{1}{Q}+\frac{2}{A_0}\right) \longrightarrow Q\cdot\left(\frac{1}{Q}-\frac{1}{Q'}\right) = \frac{Q'-Q}{Q'Q}\cdot Q \simeq \frac{\Delta Q}{Q} = -\frac{2Q}{A_0} \qquad \text{IN-BAND DROOP}$$

$\longrightarrow$ FINITE GBWP EFFECT

Let's write now the real integrator T.F. at high frequency:

$$H_{int}''(s) = -\frac{w_0}{s}\frac{1}{\left(1+\frac{s}{w_M}\right)}$$

so we get

$$T(s) = \frac{\left(\frac{w_0}{s}\right)^2\cdot\frac{1}{\left(1+\frac{s}{w_M}\right)^2}}{\left(\frac{w_0}{s}\right)^2\frac{1}{\left(1+\frac{s}{w_M}\right)^2}+\frac{w_0}{Qs}\frac{1}{\left(1+\frac{s}{w_M}\right)}+1}$$

$$= \frac{1}{1+\frac{s}{Qw_0}\left(1+\frac{s}{w_M}\right)+\frac{s^2}{w_0^2}\left(1+\frac{s}{w_M}\right)^2}.$$

This T.F. has two complex conjugate poles + two additional poles due to the integrator. Assuming to work at $|s| << w_M$ we may write

$$\left(1+\frac{s}{w_M}\right)^2\left[\frac{s^2}{w_0^2}+\frac{s}{Qw_0\left(1+\frac{s}{w_M}\right)}+\frac{1}{\left(1+s/w_M\right)^2}\right] = 0$$

$$\Downarrow \simeq$$

$$\left(1+\frac{s}{w_M}\right)^2\left[\frac{s^2}{w_0^2}+\frac{s}{Qw_0}\left(1-\frac{s}{w_M}\right)+\left(1-\frac{s}{w_M}\right)^2\right] = 0$$

so we get

$$\frac{s^2}{w_0^2}\left[1-\frac{w_0}{Qw_M}+\left(\frac{w_0}{w_M}\right)^2\right]+\frac{s}{w_0}\left(\frac{1}{Q}-2\frac{w_0}{w_M}\right)+1=0$$

$$\downarrow$$

$$s^2+s\frac{w_0}{\left[1-\frac{w_0}{Qw_M}+\left(\frac{w_0}{w_M}\right)^2\right]}\frac{\left(1-2\frac{w_0}{w_M}Q\right)}{Q}+\frac{w_0^2}{\left[1-\frac{w_0}{Qw_M}+\left(\frac{w_0}{w_M}\right)^2\right]}=0$$

$$\Longrightarrow\quad w_0'=\frac{w_0}{\sqrt{1-\frac{w_0}{Qw_M}+\left(\frac{w_0}{w_M}\right)^2}}\simeq w_0\left(1+\frac{w_0}{2Qw_M}\right)\qquad(w_M>>w_0)$$

$$\Longrightarrow\quad \frac{1}{Q'}\simeq\frac{1}{Q}\frac{\left(1-\frac{2w_0}{w_M}Q\right)}{\left[1-\frac{w_0}{Qw_M}+\left(\frac{w_0}{w_M}\right)^2\right]}\simeq\left(\frac{1}{Q}-\frac{2w_0}{w_M}\right)\qquad(w_M>>w_0)$$

So $\dfrac{Q'-Q}{Q'}\simeq\dfrac{\Delta Q}{Q}=\dfrac{2w_0}{w_M}\cdot Q$

CONCLUSION: the real response of the opAMP affects the w_0 and the Q of the real integrator, so eventually it affects the gain of the filter, which becomes more sensitive if the Q factor is higher! This happens because the $\frac{\Delta Q}{Q}$ terms are proportional to Q.
Although the positive Q shifts due to HF singularities may compensate the negative shift due to the DC gain and the LF pole, we consider the worst case, setting a constraint on the maximum absolute value for both the positive and negative swing.

$$\left|\frac{\Delta Q}{Q}\right|\leq \varepsilon_{ERR}\%$$

As a final result, we may consider the contribution arising from the positive zero in the open loop Op-AMP T.F., so

$$H_{int}(s)=-\frac{w_0}{s}\left(1-\frac{s}{w_Z}\right)$$

$$T(s)=\frac{H_{int}^2(s)}{H_{int}^2(s)-\frac{1}{Q}H_{int}(s)+1}=\frac{\left(\frac{w_0}{s}\right)^2\left(1-\frac{s}{w_Z}\right)^2}{\left(\frac{w_0}{s}\right)^2\left(1-\frac{s}{w_Z}\right)^2+\frac{w_0}{Qs}\left(1-\frac{s}{w_Z}\right)+1}$$

The poles will be given by the roots of

$$\frac{s^2}{w_0^2}+\frac{s}{Qw_0}\left(1-\frac{s}{w_Z}\right)+\left(1-\frac{s}{w_Z}\right)^2=0$$

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So we get

$$\frac{s^2}{\omega_0^2}\left[1-\frac{\omega_0}{Q\omega_z}+\left(\frac{\omega_0}{\omega_z}\right)^2\right]+\frac{s}{Q\omega_0}\left(1-2\frac{\omega_0}{\omega_z}Q\right)+1=0$$

$$\downarrow$$

$$s^2+\frac{s\,\omega_0}{\left[1-\frac{\omega_0}{Q\omega_z}+\left(\frac{\omega_0}{\omega_z}\right)^2\right]}\,\frac{\left(1-2\frac{\omega_0}{\omega_z}Q\right)}{Q}+\frac{\omega_0^2}{\left[1-\frac{\omega_0}{Q\omega_z}+\frac{\omega_0}{\omega_z}\right]^2}=0$$

So

$$\omega_0'=\frac{\omega_0}{\sqrt{1-\frac{\omega_0}{Q\omega_z}+\left(\frac{\omega_0}{\omega_z}\right)^2}}\simeq\omega_0\left(1+\frac{2\omega_0}{Q\omega_z}\right)\qquad(\omega_z>>\omega_0)$$

$$\frac{1}{Q'}=\frac{1}{Q}\,\frac{\left(1-2\frac{\omega_0}{\omega_z}Q\right)}{\sqrt{1-\frac{\omega_0}{Q\omega_z}+\left(\frac{\omega_0}{\omega_z}\right)^2}}\simeq\frac{1}{Q}\left(1-\frac{2\omega_0}{\omega_z}\right)\qquad(\omega_z>>\omega_0)$$

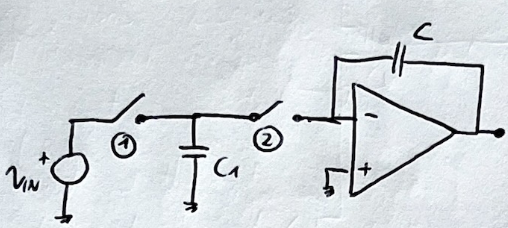
$$\Downarrow$$

$$\left(\frac{1}{Q}-\frac{1}{Q'}\right)Q\simeq\frac{\Delta Q}{Q}=\frac{2\omega_0}{\omega_z}Q$$

POSITIVE SHIFT, like for the GBWP

SWITCHED CAPACITORS

In the low frequency range we'd require very large resistors in order to have suitable values of capacitors, but they are hard to be made in integrated technology, so we use switched capacitors



Phase 1 $\Rightarrow$ 1 is closed and C_1 stores $Q = C_1 \cdot v_{IN}$

Phase 2 $\Rightarrow$ 2 is closed and C_1 charges C, so $v_C = \frac{C_1 V_{IN}}{C}$

At the output we get a staircase with slope $\frac{C_1 V_{IN}}{C \cdot T}$

The output of an integrator has a slope of

$$\frac{V_{IN}}{RC}$$

so we can say that the switched capacitor provides $R_{eq} = \frac{T}{C_1}$

ADVANTAGES:
- makes large resistors by very small capacitors
- no need of buffer output stages in ladder networks made of integrators
- ω_0 depends on the ratio between two capacitors, which can be controlled very well

DISADVANTAGES: we have to deal with a discrete signal, so Shannon must be respected

ANALYSIS OF A CASCODE - COMPENSATED OTA

A1

The following is an idea to compensate a two-stage OTA in an Ahuja-like technique but without using too much current.

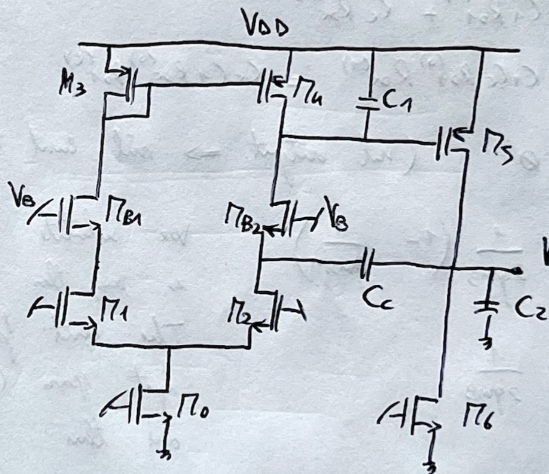
IDEA: substitute the current buffer with a cascode in the first stage.

The T.F. will have this general form:

$$T(s) = G(0) \cdot \frac{a_2 s^2 + a_1 s + 1}{b_3 s^3 + b_2 s^2 + b_1 s + 1}$$

because we have three independent capacitors and one of them is directly attached to the output node from ground.

We find that:



$$G(0) = g_{m1} R_{OUT,1}\, g_{m5} R_{OUT,2}$$

$$b_1 = C_1 R_1^{(0)} + C_2 R_2^{(0)} + C_C R_C^{(0)}$$

$$b_2 = C_1 C_2 R_1^{(0)} R_2^{(1)} + C_1 C_C R_1^{(0)} R_C^{(1)} + C_2 C_C R_2^{(0)} R_C^{(2)}$$

$$b_3 = C_1 C_2 C_C R_1^{(0)} R_2^{(1)} R_C^{(1,2)}$$

so for the denominator we need

$$R_1^{(0)} = R_1$$

$$R_2^{(0)} = R_2$$

$$R_2^{(1)} = R_2$$

$$R_C^{(1)} = \frac{1}{g_{mB}} + R_2 \simeq R_2$$

$$R_C^{(2)} = \frac{1}{g_{mB}}$$

in order to compute $R_C^{(0)}$ we may substitute C_C with a current generator i_S.

This current will flow basically entirely through M_{B2}, thus generating:

$$v_{gs} = i_S R_1$$

so we get the following balance at the output node:

$$i_S + g_{m5} R_1 i_S + \frac{v_2}{R_2} = 0$$

$$\Downarrow$$

$$v_2 = -\, i_S R_2 (1 + g_{m5} R_1)$$

Then we have that

$$v_1 \simeq \frac{i_S}{g_{mB}}$$

so we get

$$v_S = v_1 - v_2 = \left(\frac{1}{g_{mB}} + R_2 + g_{m5} R_1 R_2\right) i_S$$ <u>MILLER EFFECT</u>

Therefore we get

$$R_C^{(0)} \simeq g_{mS} R_1 R_2$$

and we can safely say that $b_1 \simeq C_c \, g_{mS} R_1 R_2$

Thus, we have that

$$b_2 \simeq C_1 C_2 R_1 R_2 + C_1 C_c R_1 R_2 + C_2 C_c R_2 \cdot \frac{1}{g_{mB}} \simeq C_1 R_1 R_2 (C_2 + C_c)$$

$$b_3 = C_1 C_2 C_c \, R_1 R_2 \frac{1}{g_{mB}}$$

Let's consider now the numerator of the T.F.:

$$a_1 = C_1 R_{o1}^{(0)} + C_c R_{oc}^{(0)}$$

$$a_2 = C_1 C_c R_{o1}^{(0)} R_{oc}^{(1)} = C_c C_1 R_{oc}^{(0)} R_{o1}^{(c)}$$

$R_{o1}^{(0)} = 0$ (nil output $\Rightarrow$ nil current through M_S $\Rightarrow$ nil $v_{g,S} = v_S$)

$$R_{oc}^{(0)} = \frac{1}{2 g_{mB}} \left(1 - \frac{1}{g_{mS} R_1}\right) \Big\downarrow \simeq \frac{1}{2 g_{mB}}$$

We substitute C_c with a current source. If the output is zero, then i_S flows into M_S, so $v_{g,S} = -\frac{i_S}{g_{mS}}$. The current from i_S merges with the one due to the input pair (splits), so we have $i_S - i_d$ through M_{B2} and thus

$$i_S - i_d - (i_d) = i_S - 2 i_d$$

through R_1. So we get

$$-\frac{i_S}{g_{mS} R_1} = i_S - 2 i_d$$

$$\Downarrow$$

$$i_d = \frac{i_S}{2} \left[1 + \frac{1}{g_{mS} R_1}\right]$$

so the voltage at the source of M_{2B} will be

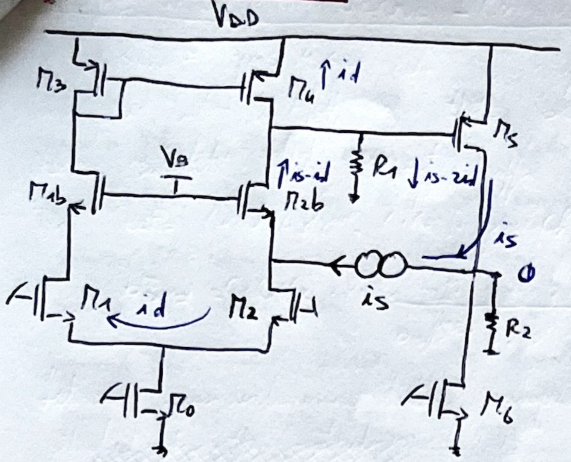
$$\frac{(i_S - i_d)}{g_{mB}} = \frac{1}{g_{mB}} \left[\frac{i_S}{2} - \frac{1}{g_{mS} R_1} \frac{i_S}{2}\right] = v_S$$

$$\Longrightarrow \frac{v_S}{i_S} = \frac{1}{2 g_{mB}} \left[1 - \frac{1}{g_{mS} R_1}\right]$$

Now, since $R_{o1}^{(0)} = 0$, it's better to use the second form, in order not to get indeterminate results:

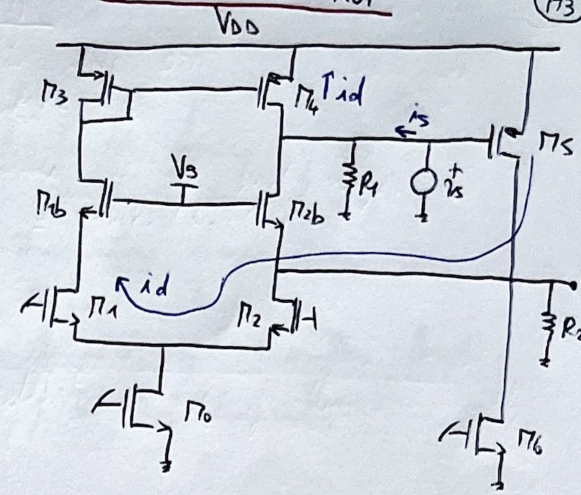
$$i_d = i_S = -g_{mS} v_S$$

COMPUTATION OF $R_{oc}^{(c)}$



COMPUTATION OF $R_{o1}^{(c)}$

A3



Then $i_s = i_d + \frac{v_s}{R_1} \simeq i_d = -g_{m5} v_s$

so

$$R_{o1}^{(c)} \simeq -\frac{1}{g_{m5}}$$

in the end we have

$$a_1 \simeq \frac{C_c}{2g_{mB}}$$

$$a_2 = C_c C_1 \frac{1}{2g_{mB}}\left(-\frac{1}{g_{m5}}\right)$$

and

$$T(s) \simeq g_{m1} g_{m5} R_1 R_2 \cdot \frac{-s^2 \frac{C_c C_1}{2g_{mB} g_{m5}} + s\frac{C_c}{2g_{mB}} + 1}{C_1 C_2 C_c \frac{R_1 R_2}{g_{mB}} s^3 + C_1 R_1 R_2 (C_2 + C_c) s^2 + g_{m5} C_c R_1 R_2 s + 1}$$

DESIGNERS RULES OF THUMB

B1

- → The common mode swing sets overdrive constraints for the input stage
- → The output swing sets constraints for the overdrive of the output stage
- → SR, GBWP and E^2_{in} set constraints on the current of the input stage
- → The gain sets constraints on the output transistors length
- → The offset sets constraints on the input transistors length
- → The noise corner sets constraints on the nMOS input transistors length
- → CMRR sets constraints on the length of the tail transistor

ROOT LOCUS RULES

- # BRANCHES = # POLES OF $G_{loop}(s)$;
- Branches proceed from poles to zeroes (finite or infinite);
- $\gamma < 0$ regions on the $\mathrm{Re}[\,]$ axis on the left of an odd number of singularities;
- $\gamma > 0$ regions of the $\mathrm{Re}[\,]$ axis on the left side of an even number of singularities;
- # ASYMPTOTES = (#POLES) − (# ZEROES @ FINITE FREQUENCY)
- ASYMPTOTES = branches going to ∞ that always split the Gauss plane into even parts

Write the T.F. in the Evans form.

(I)NSIGHTS FROM SWITCHED CAPACITORS DESIGN

The switched capacitor (SC) mimics the behaviour of a large resistance, so it is widely used in filter design in audio range. We can make integrators based on a SC, who basically integrates current pulses, hence providing voltage steps at the output. The result is a staircase output waveform with an equivalent ramp-rate

$$\frac{\Delta V}{T} = \frac{E \cdot C_1}{C} \cdot \frac{1}{T} = \frac{E}{C \cdot R_{eq}}$$

$$R_{eq} = \frac{T}{C_1}$$

The discrete approximation is good enough provided that the clock frequency is much larger than the BW of the input signal, hence it respects Shannon theorem. This can be easily done, since typical f_{CK} are $\simeq 1\,MHz$, while the audio range is $\simeq 20\,kHz$ at the most.

We've already discussed the advantages, let's see now what's the bad part of dealing with discrete time systems:

we can write the output waveform like a staircase

$$v_{OUT}(t) = \sum_{m=0}^{+\infty} v_{OUT}(mT) * \left\{ \text{rect}\left[\frac{t - mT}{T}\right] \right\}$$

where

$$\text{rect}(x) = \begin{cases} 1 & |x| < \frac{1}{2} \\ 0 & |x| > \frac{1}{2} \end{cases}$$

and

$$\mathcal{F}[\text{rect}(t)](f) = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-j2\pi f t}\, dt = -\frac{1}{2\pi f j}\left[e^{-j2\pi f t} \right]_{-\frac{1}{2}}^{\frac{1}{2}} =$$

$$=$$

$$= \frac{1}{\pi f} \cdot \frac{1}{2j}\left[e^{j\pi f} - e^{-j\pi f} \right] = \frac{\sin(\pi f)}{\pi f} = \text{sinc}(f)$$

Let's see now what is the link between the Fourier transform of the continuous-time signal and the discrete-time one, from input to output:

$$\mathcal{F}[v_{OUT}(t)] = \sum_{m=0}^{+\infty} v_{OUT}(mT) \cdot \underbrace{e^{-j\omega mT}\, \text{sinc}\left(\frac{f}{f_T}\right) \cdot T}_{\mathcal{F}\left\{\text{rect}\left[\frac{t - mT}{T}\right]\right\}} = \underbrace{\sum_{m=-\infty}^{+\infty} v_{OUT}(mT)\, z^{-m} \Big|_{z = e^{j2\pi f t}}}_{\text{ZETA TRANSFORM}} \cdot T\, \text{sinc}\left(\frac{f}{f_T}\right)$$

(C2)

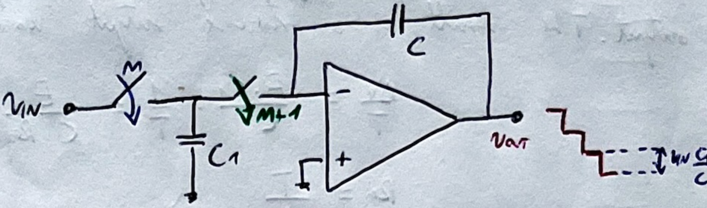
So we came to this result:

$$V_{OUT}(f) = V_{OUT}(z)\Big|_{z=e^{j2\pi f t}} \cdot T \operatorname{sinc}\left(\frac{f}{T}\right)$$

Now, from the circuit we have that

$$v_{OUT}(n+1) = v_{OUT}(n) - v_{IN}(n)\frac{C_1}{C}$$

$\Downarrow$ ZETA TRANSFORM



$$V_{OUT}(z)\cdot z = V_{OUT}(z) - V_{IN}(z)\frac{C_1}{C}$$

$\downarrow$

$$\frac{V_{OUT}}{V_{IN}}(z) = -\frac{C_1}{C}\frac{1}{z-1} = H(z) \Longrightarrow$$ Transfer function of the discrete-time filter

So we can write

$$V_{OUT}(f) = V_{IN}(z)\cdot H(z)\Big|_{z=e^{j2\pi f t}} \cdot T \operatorname{sinc}\left(\frac{f}{T}\right)$$

where

$$V_{IN}(z)\Big|_{z=e^{j2\pi f t}} = \sum_{n=0}^{+\infty} v_{IN}(nT)\, e^{-j2\pi f t n} = \int_{-\infty}^{+\infty} v_{IN}(t)\cdot\sum_{n=0}^{+\infty}\delta(t-nT)\, e^{-j2\pi f t}\, dt$$

$$= \mathcal{F}\left[v_{IN}(t)\sum_{n=0}^{+\infty}\delta(t-nT)\right](f)$$

$$= V_{IN}(f) * \sum_{k=-\infty}^{+\infty}\frac{1}{T}\,\delta\left(f-\frac{k}{T}\right)$$

and, in the end, we have

$$V_{OUT}(f) = \underbrace{\left[V_{IN}(f) * \sum_{k=-\infty}^{+\infty}\frac{1}{T}\,\delta\left(f-\frac{k}{T}\right)\right]}_{\text{aliasing due to sampling}}\cdot\underbrace{\left[-\frac{C_1}{C}\,\frac{1}{e^{j2\pi f t}-1}\right]}_{\text{action of the integrator, is a periodic filter}}\cdot\underbrace{\left[T\operatorname{sinc}\left(\frac{f}{T}\right)\right]}_{\text{Spurious cardinal sine due to sampling}}$$

In order to have a reliable output, we need:

→ a RECONSTRUCTION FILTER that kills all the HF harmonics due to aliasing;

→ an ANTI-ALIASING FILTER before the the SC filter in order to kill HF disturbances that may be brought to base-band due to aliasing;

→ an EQUALIZING FILTER that compensates the spectrum shape alteration due to the cardinal sine term.

All these filters can be easily implemented by continuous-time networks